

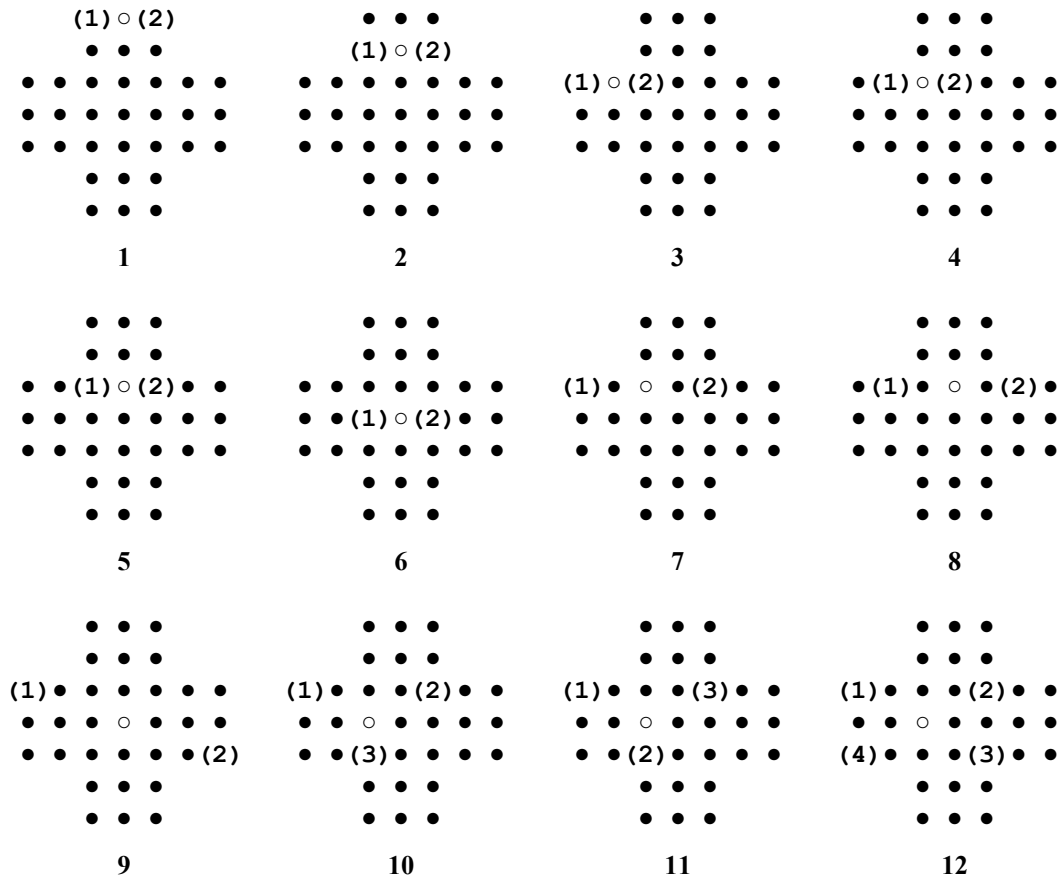
### 33-hole solitaire : Swaps and merry-go-rounds

JDB, 13 July 2013

Although I looked briefly at merry-go-round problems on page 156 of *The Ins and Outs of Peg Solitaire*, I did so only in the context of “man-on-the-watch” problems, where the marked men remained stationary until the final loop. Also of interest, and often more difficult, are problems where an interchange or cycle of men is possible but only if one or more of them is allowed to move earlier in the play (as in **10.49** in *The Ins and Outs*). Adding these to the “man-on-the-watch” merry-go-rounds gives the attractive set of problems shown below. In each case the task is to interchange or cycle the men numbered **1**, **2**, etc, and the initial vacancy is midway between them. Where the cycle involves more than two men, it is to be performed in the direction  $1 \rightarrow 2 \rightarrow \dots \rightarrow 1$  (so problems **10** and **11**, although involving the same men, are different), but it is not necessarily man **1** which moves first.

Notation:

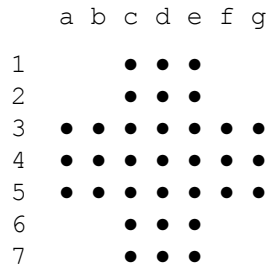
- a hole empty at the start of play
- a hole occupied by an unmarked man at the start of play
- ( ) a hole to be occupied at the end of play (all other holes are to be left empty)
- m a hole occupied by a marked man at the start of play (these men are to cycle 1, 2, etc)



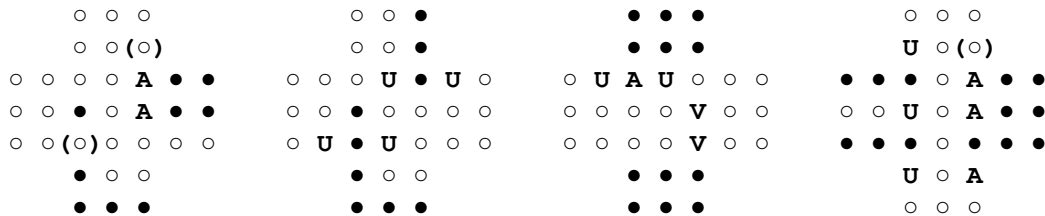
Solutions on following pages.

## Solutions

For individual moves, we shall use an algebraic notation



and chess players are asked to note that in solitaire we put row 1 at the top. We shall also denote the outside corners of the board by “top left” (c1), “top right” (e1), “right upper” (g3), “right lower” (g5), and so on round. Furthermore, several short sequences of moves are frequently repeated, and it is convenient to both writer and reader to have a shorthand for them. Readers already familiar with these sequences, whether through reading *The Ins and Outs* or otherwise, should skip the rest of this page.



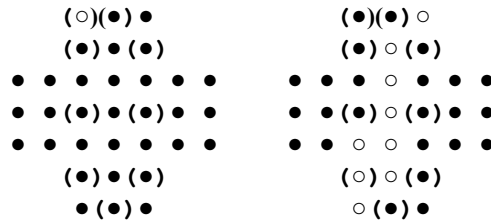
In the first diagram, the bottom left-hand corner shows the **L-move**: play c7-c5, c4-c6, e7-c7-c5. We shall denote this by “L-move into c5”. The right upper corner shows the **four-move**: in some order, play e4-e2, g3-e3, g4-e4, the precise order depending on whether the holes AA are full or empty at the start. The effect of this combination is to replace the four men in f3/g3/f4/g4 by a single man in e2, and to put the contents of the two holes AA back as they were at the start of the sequence (it doesn't matter how they started, there is always an order in which we can play the moves, and it always restores them to their original state). We shall denote this combination by “four-move into e2”.

The top three rows of the second diagram show the **three-removal**. If the contents of holes UU are unlike, a horizontal move across e3 will be possible, so play this move, then a move e1-e3, and then a horizontal move back across e3, removing the three men e1/e2/e3 and putting the holes UU back as we found them. We shall denote this by “three-removal e1/e2/e3”, or by “three-removal top right” where, as here, it is used to clear an outside corner. The bottom four rows show the **L-removal**: the contents of holes UU being unlike, play a horizontal move across c5, an L-move into c5, and a horizontal move back. We shall call this “L-removal bottom left”.

The third diagram shows the **six-removal**. At the top, the contents of UU being unlike and A being empty, play c1-c3, a horizontal move across c3, e2-c2, e1-c1-c3, and a move back across c3; if A is full, the sequence is horizontal move across c3, c1-c3, move back across c3, e2-c2, e1-c1-c3. In either case, the top six is cleared and the contents of holes UAU are restored to their original state. At the bottom, the sequences are similar except that the moves across e5 are vertical and it is holes VV whose contents must be unlike.

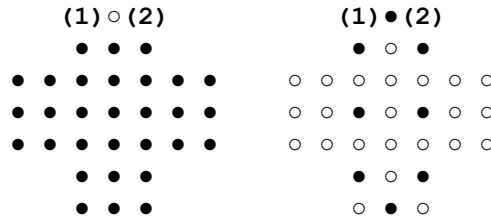
The left-hand side of the fourth diagram shows the **double-three**. Provided that the contents of any two of the holes UUU are unlike, the two sets of three a3/b3/c3 and a5/b5/c5 can be removed. For example, suppose c2 empty and c4/c6 full. We have the conditions for a three-removal a3/b3/c3, so we play c4-c2 to start this, but now we have the conditions for a second three-removal a5/b5/c5, so we break off the first three-removal, play the second, and then resume and complete the first. As oriented here, we shall call this “double-three left”. The right-hand side shows the **four-three**, which is similar. Whatever the state of the three holes AAA, we can start playing a four-move f3/g3/f4/g4 into e2, break off at a suitable moment, play the three-removal e5/f5/g5, and then resume and complete the four-move. We shall call this “four-three into e2”.

To see some of these sequences in action, consider the problem “vacate c1 and play to finish there” with the added condition that the final move be a loop of eight, a problem by Bergholt given as 9.9 in *The Ins and Outs*. In our notation, the problem is shown in the left-hand diagram below,

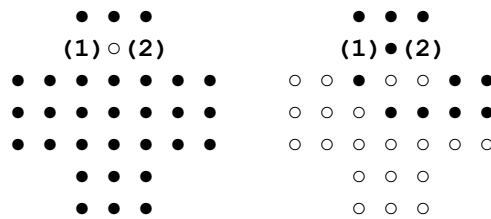


and the solution proceeds e1-c1, d3-d1, d5-d3, b5-d5, d6-d4, c7-c5, d3-d5-b5 (now we have the diagram on the right), four-three on the left into c6 (this clears a3/b3/c3 as well as filling c6), three-removal at bottom right, four-three into e6, and we are ready to go.

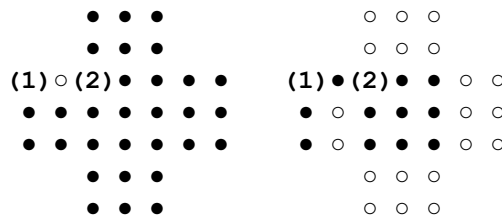
Sets of squares such as UU and VV are called *catalysts* by John Conway; if their states of occupancy are different, they make an operation possible while being themselves unchanged by it. However, while the state of occupancy of squares such as VV and AA is restored by these operations, the original men in them are normally not preserved, and care must always be taken to perform these operations without disturbing the marked men. A marked man may in fact be part of a UU catalyst, unless a “man on the watch” solution is being sought, but in none of these solutions shall we so use it.



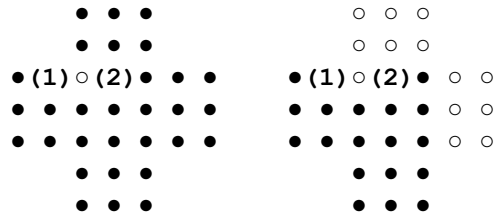
Problem 1 yields to almost exactly the same play as the Bergholt “loop of eight” problem given above; all we have to do is to omit the first move. Now we finish with the marked men still at c1/e1 and unmarked men at c2/c4/c6/d7/e6/e4/e2/d1 (see right-hand diagram), and the marked men cycle round.



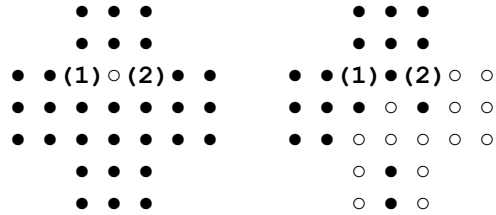
For problem 2, play d4-d2, b3-d3, e3-c3, b4-d4, L-removal left lower, six-removal bottom, three-removal right lower (now we have the right-hand diagram), c2-c4, e2-c2, four-move into e2, L-move into e3, c4-e4-e2.



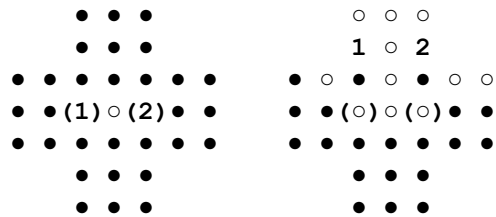
For problem 3, b5-b3, six-removal bottom, six-removal right, six-removal top (right-hand diagram), three-removal e3/e4/e5, L-move into b5 (not a usual location for an L-move, but so what), and round we go.



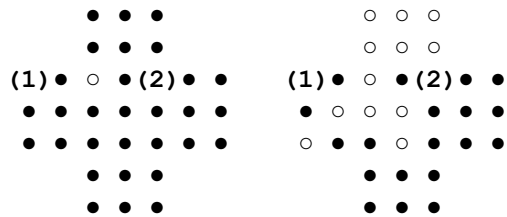
Problem 4, six-removal top, six-removal right (right-hand diagram), d4-f4, six-removal bottom, c5-c3, L-move into c5, and an asymmetrical six-chain finish. If a symmetrical four-chain finish is preferred, replace the move d4-f4 by a three-removal e3/e4/e5.



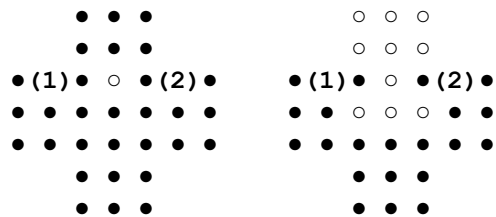
Problem 5, d5-d3, six-removal right, double-three bottom (right-hand diagram), d7-d5, c3-c5-e5, six-removal left, six-removal top, e3-c3, e5-e3.



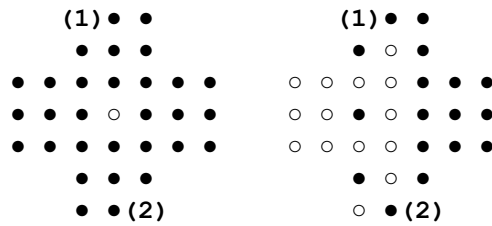
Problem 6, d2-d4, b3-d3, c1-c3, c2-c4, d4-d2, d1-d3, e3-c3, e1-e3, e4-e2, g3-e3 (now the marked men are safely out of the way, see right-hand diagram), three-removal right lower, g4-e4, L-move into e5, c6-c4, L-move into c5, b4-d4, c2-c4-c6, e2-e4-c4, c6-e6-e4. This is 10.49 in *The Ins and Outs*.



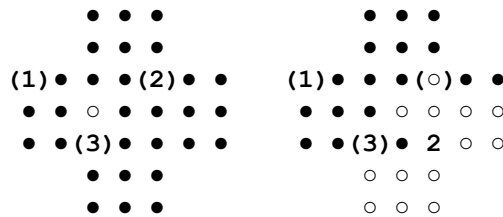
Problem 7, six-removal top, c5-c3, a5-c5, b3-b5, d3-b3, d5-d3 (right-hand diagram), double-three bottom, d7-d5, six-removal right.



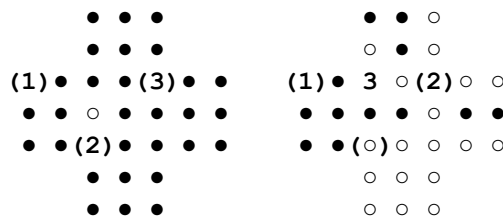
Problem 8, d1-d3, d4-d2, three-removal e2/e3/e4, c2-e2, e1-e3, c4-c2, c1-c3 (right-hand diagram), L--removal right lower, e7-e5, d7-d5, L-removal left lower, c7-c5. This is 1.14 in *The Ins and Outs*.



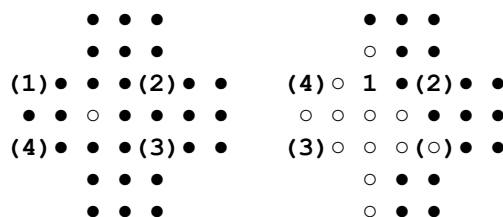
We reset problem 9 in the equivalent form “vacate d4 and play to interchange c1 and e7” (left-hand diagram) in order to bring out the similarity with problem 1 (vacate d1 and play to interchange c1 and e1). Play d2-d4, and then follow the solution to problem 1 as far as the four-three into c6. This gives the right-hand diagram, and now, instead of a three-removal at the bottom right and a four-three into e6, play the three-removal at the top right and the four-three into e2. In each case, we are left with an eight-loop with the marked men somewhere on it, and they can cycle round.



Problem 10, e4-c4, three-removal right lower, six-removal bottom, g4-e4, e3-e5 (right-hand diagram), g3-e3, double-three top, a3-c3-e3, b5-b3, a5-a3-c3, d1-d3-b3, c5-c3-a3, e5-c5.



Problem 11, c2-c4, e3-c3, e1-e3, double-three right, six-removal bottom, c5-e5-e3 (right-hand diagram), a5-c5, a3-a5, c3-a3, c5-c3, g4-e4-c4-c2, c1-c3, d1-d3-b3-b5, a5-c5. This and problem 10 are probably the hardest of the set, though problem 6 is not far behind.



Problem 12, c2-c4, a3-c3, b5-b3, three-removal bottom left, d4-b4, b3-b5, a5-a3, e5-c5-a5 (right-hand diagram), d2-d4, d7-d5-d3, e7-e5, three-removal right lower, e3-e5, g3-e3, g4-e4, L-removal top right, c3-e3. To cycle round the other way, reflect this solution top to bottom.

For problems of this kind on other boards, see problem 1 in issue 28 of *The Games and Puzzles Journal* (37-hole board) and chapter 9 of *51 flights of chess fancy* (various 13-hole boards) As regards the 33-hole board, I think these are all the problems of this kind which have solutions (the four-cycle which is 10.23 in *The Ins and Outs* does not have the initial vacancy in the middle of the square), but some of the proofs involve writing out move maps and verifying that none of them can be executed, and at 73 my capacity for oversight is considerable. I am therefore reluctant to be dogmatic, but if anyone finds a solution to another problem of this type please will he or she bring it to my attention.