Some observations on *The Ins and Outs of Peg Solitaire*

John Beasley, October 2021

The Ins and Outs of Peg Solitaire has now been out for 36 years, and the recent appearance of *The Delights of Peg Solitaire* makes this a good moment to review it and to consider what has changed in the interim.

History

Whatever I may have thought at the time, the treatment of the game's history in *The Ins and Outs* is now seen to be almost embarrassingly incomplete and misleading; almost the only thing that still stands up is the pouring of cold water on the legend of the prisoner in the Bastille. There is no point in going through the text in detail; it is sufficient to say that the historical material in *The Delights* totally supersedes that in *The Ins and Outs*.

Problems

Inevitably, a number of good problems have come to light since *The Ins and Outs* was written. Most of those known to me are in *The Delights*; the following, whose solution requires the use of fractional jumps, is not.

X					○	○
						○
○						○
○						○
○						○
○						
○	○					X

The aim is to reduce the twelve men to two men in the holes marked X, and with normal play the task is quickly seen to be impossible. However, let us divide the man at g1 into two portions in the ratio 5:3, and send the larger portion leftwards and the smaller portion downwards. Leftwards, we have 5/8 into e1 leaving 3/8 in f1; 3/8 into d1 leaving 2/8 in e1; 2/8 into c1 leaving 1/8 in d1; 1/8 into b1 leaving 1/8 in c1; 1/8 into a1. Downwards, 3/8 into g3 leaving 5/8 in g2 and making 11/8 in g3 (if this putting of more than one man into a hole is thought objectionable, the jumps can be divided and reordered to avoid it); 5/8 into g4 leaving 6/8 in g3 and making 13/8 in g4; 6/8 into g5 leaving 7/8 in g4 and making 14/8 in g5; 7/8 into g6 leaving 7/8 in g5; 7/8 into g7. Do the equivalent with the man at a7, and the problem is solved.

This would have made an excellent example for the section on Fractional Solitaire. It generalizes, of course; on an $n \times n$ square with $n \geq 5$, the men in the corners must be divided in the ratio $F_{n-2} : F_{n-3}$ where F_n is the nth Fibonacci number ($F_0 = 0$, $F_1 = 1$, $F_{n\geq 2} = F_{n-2} + F_{n-1}$). It also works on a ring, perhaps even more strikingly. Suppose a ring of $2n$ holes numbered in clockface style, n at the bottom and $2n$ at the top, with men in holes 2 to n+1 inclusive; we can now get a man into the apparently inaccessible hole $2n$ by dividing the man in hole n in the ratio $F_{n-1} : F_{n-2}$ and sending one part each way.

Notations for future work

If I were writing *The Ins and Outs* today, I would change to the notations used in *The Delights* with one exception: if I were writing from a mathematical standpoint, I would continue to use "complement" instead of "reversal". I would also use "weighted resource count" instead of the simple "resource count". I am aware that the originators of the idea used the enjoyably and even gloriously picturesque term "pagoda function", but in truth there is nothing remotely pagoda-like about most of its applications, and the use of a term so exotic inevitably gives the impression that something deep and subtle is going on whereas in truth the matter is very simple. "Weighted resource count" may be sadly banal in comparison, but it indicates precisely the sort of thing that is being measured.

Thoughts on a reprint

Nobody has suggested a reprint to me, and I hardly think anyone is likely to. If the question were to arise, I would be very happy, but if we take the 1992 edition as our reference point I would want page 252 to be replaced by an updated summary of historical knowledge, with references to *The Delights* and to anything else that may come to light in the meantime. If the Fractional Solitaire problem above could be squeezed in as well, so much the better.