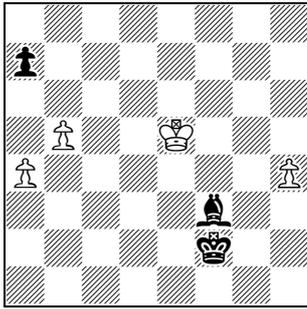


The Chess Endgame Studies of Richard Réti : Bishops and pawns

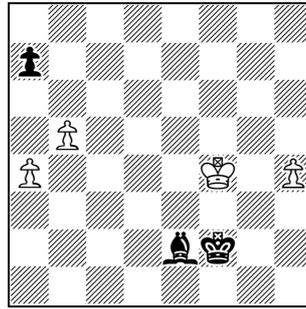
John Beasley, 14 January 2012, latest revision 20 January, minor correction 2 March

3.1 (M 10)



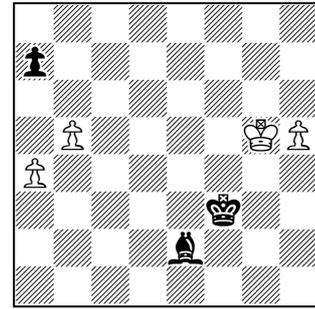
White to play and win

3.1a



1 Kf4, after 1...Be2

3.1b



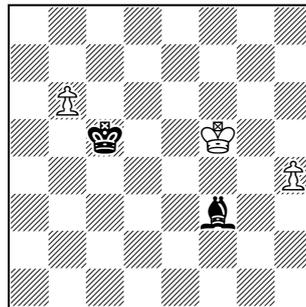
Main line, after 4 h5

Put Black's king on the first rank in **3.1** (3rd Prize, *Kagan's Neueste Schachnachrichten*, 1922) and his bishop will soon be overloaded (1 a5 K~ 2 b6 axb6 3 axb6 K~ 4 h5 etc). However, as it stands he threatens 1...Kg3 and 2...Kxh4, after which the bishop will be stop the queen's side pawns. The obvious move for White is therefore 1 Kf4, but 1...Be2 turns out to put him in zugzwang (see **3.1a**). The pawns are paralysed, if 2 Kg5 then 2...Ke3 ready to take over the defence of the queen's side (3 h5 Bxh5 4 Kxh5 Kd4 etc), if 2 Kf5 then 2...Ke3 3 Ke5 Kd2 4 Kd4 Kc2 5 Kc5 Kd3 6 a5 Ke4 catching the h-pawn, if 2 Ke5 then 2...Kg3 as previously, and if 2 Ke4 then 2...Kg3 3 Ke3 Bg4 4 a5 Kxh4 5 b6 axb6 6 axb6 Bc8.

The solution is **1 Kf5**, when 1...Kg3 can be met by 2 Kg5 (2...Be2 3 h5 with 4 h6 and 5 a5), and if again **1...Be2** to paralyse the pawns then **2 Kf4** and we have **3.1a** with Black to play. A bishop move will release one of the pawns, and a king move to the bottom rank will put him too far from the action (2...Ke1 3 Kg5 Kd2 4 h5 Kc3 5 h6 Bd3 6 a5). This leaves **2...Kg2**, met by the natural **3 Kg5**. 3...Kg3 and 3...Kf2 are now too slow (4 h5 Bxh5 5 Kxh5 K~ 6 a5 and wins), and **3...Kf3** blocks the bishop and again allows **4 h5** (see **3.1b**). If 4...Bxh5 were now possible it would draw (5 Kxh5 Ke4 etc), but it isn't, and an immediate 4...Ke4 doesn't work either (5 h6 and the bishop's path to h7 is blocked). Black has nothing better than **4...Ke3**, and **5 h6 Bd3 6 a5** duly wins.

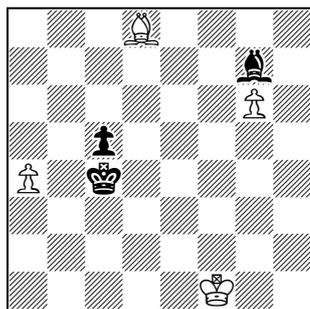
And if Black tries **1...Ke3**, Mandler gives **2 a5 Kd4 3 b6 axb6 4 axb6 Kc5** (see **3.1c** below) and now not 5 h5 (5...Kxb6 6 h6 Bd1) but **5 Kf4**, and if **5...Bd5** then **6 Ke5**.

3.1c



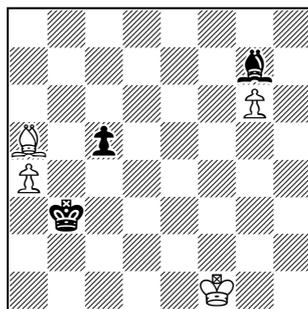
1...Ke3, after 4...Kc5

3.2 (M 12)



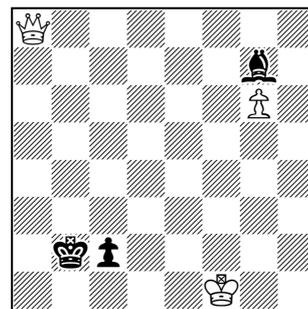
White to play and win

3.2a



After 1...Kb3

3.2b



2...Kxc3, after 6...c2

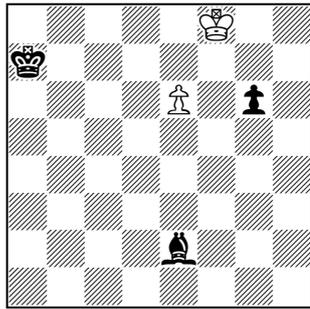
If White's bishop were on say e1 in **3.2** (*Tagesbote*, 6 September 1925, correction), he would have a routine win (pawn to a5 tying Black's king to the top left quadrant, own king to c2, bishop to c3 blocking any advance of Black's pawn and forcing Black's bishop to f8 or h6, king to f7 pushing Black's bishop to h6, and own bishop to d2 finally forcing Black to relinquish his grip of g7). But the bishop is not on e1, and Black threatens 1...Kb4 2 a5 Kb5 tying it to the defence of the a-pawn while it is still on the diagonal d8-a5. An advance of the c-pawn to c3 will now set up a fortress, nor can White play Bd7 and give up his a-pawn for Black's c-pawn because his king is not yet near enough to the top right corner (for example, 1 Ke2 Kb4 2 Bd7 Kxa4 3 Bxc5 Kb5 4 Ba3 Kc6 5 Ke3 Kd7 6 Kf4 Ke8 7 Kf5 Bc3).

So White must play **1 Ba5** to prevent 1...Kb4, and now he threatens 2 Be1 winning as before. But 1 Ba5 blocks White's a-pawn, and does not **1...Kb3** kill it at once (see **3.2a**)?

In fact no: **2 Bc3!** Mandler now gives **2...Kxc3** as the main line, dismissing 2...Bxc3 with a note that after 3 a5 the Black bishop cannot catch both White pawns, but there is a little more to it than this. Let play continue 3...c4 4 a6 Bd4, and now White must sacrifice the correct pawn so that he promotes with check: 5 a7 Bxa7 6 g7 c3 7 g8Q+ is a win, but 5 g7 Bxg7 6 a7 c3 7 a8Q c2 is only drawn. Nor does it help Black to answer 5 a7 by 5...c3 and let the a-pawn promote: 6 a8Q c2 7 Qd5+ etc. I owe these lines to Harold van der Heijden's "Endgame study database IV", where 2...Bxc3 is given as the main line.

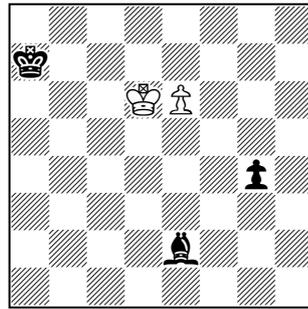
Reverting to 2...Kxc3 as given as the main line by Mandler, we have **3 a5 Kb2** (the definitive results for K + 2P v K + B + P now available show that nothing else is better) **4 a6 c4 5 a7 c3 6 a8Q c2** giving **3.2b**, and the win is not difficult. **7 Qb7+ Ka2 8 Qf7+ Ka3 9 Qc7 Kb3 10 Ke2 Bc3 11 g7** is given, and there are alternatives.

3.3 (M 13)



White to play and hold the draw

3.3a



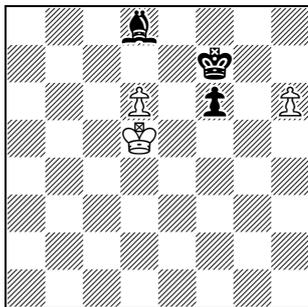
After 2...g4

In **3.3** (*Národní listy*, 10 June 1928), the theme of **1.1-1.3** is spiced by the addition of a Black bishop. Try **1 e7**: no, **1...Bb5 2 Kf7 g5** and White is dead. **1 Kf7** and **1 Kg7** are clearly no better, and the move, remarkably, is **1 Ke7** not attacking the Black pawn at all and even apparently moving away from it. If **1...Bb5** then **2 Kf6 Be8 3 Ke7 B~ 4 Kf6** drawing by attacking the Black pawn and bishop alternately, but of course Black isn't going to do this; he is going to run his pawn, **1...g5**, and after **2 Kd6** he is going to run it again, **2...g4**.

This has brought us to **3.3a**, and now is the moment for **3 e7**. The Black pawn is temporarily blocking the bishop's path to h5, so **3...Bb5** is forced, and **4 Kc5** gains a tempo by attacking it. The White king will now be able to get back and attack the Black pawn, and the bishop will be overloaded: **4...Be8 5 Kd4 Kb7 6 Ke4/Ke3 Kc7 7 Kf4 Bd7/Bh5 8 e8Q**. Nor can Black gain a tempo by playing **4...Bd7**, which I must confess I regard as Black's most natural move even though Mandler gives only ...Be8. There again follows **5 Kd4 Kb7 6 Ke4/Ke3 Kc7 7 Kf4** threatening **8 e8Q** etc, and the bishop is awkwardly preventing ...Kd7.

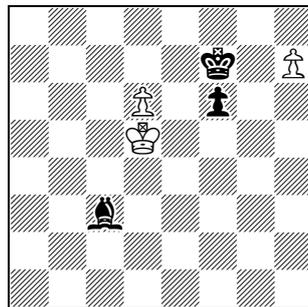
I think this has to be my favourite Réti study: elegant, ingenious, and surprising.

3.4 (M 14)



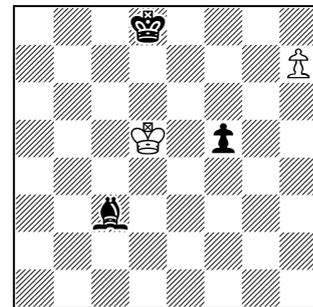
White to play and hold the draw

3.4a



After 3 h7

3.4b



After 5...Kxd8

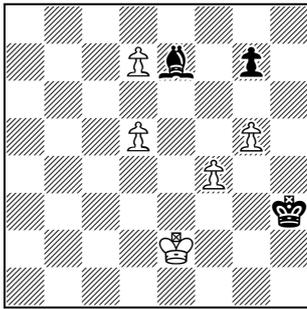
3.4 appeared as a companion piece to **3.3**. **1 h7** fails (**1...Kg7 2 Ke6 Kxh7 3 Kd7 Ba5 4 Ke6 Kg6**) and the correct move is **1 Kc6**. If **1...f5** then **2 Kd5 Bf6** (to prevent **3 Ke5**) **3 d7 Ke7 4 d8Q+ Kxd8 5 Ke6** attacking both bishop and pawn, and if **1...Kf8** then **2 Kd7 Ba5 3 Ke6 Bd8 4 Kd7** with a repetition. Hence **1...Ba5**, and now **2 Kd5** sets Black a bit of a problem. A king move will allow **3 Ke6** attacking the pawn, after which the draw will follow in a few moves, **2...Bd8** will return to the starting position, and **2...Bb6** is soon seen to do nothing useful. The only move to offer a hope of progress is **2...Bc3**, and **3 h7** gives **3.4a**.

Black now has two moves. If **3...Kg7** then **4 Ke6 (4...Kxh7 5 d7 etc)**, but not **4 d7** at once (**4...Ba5 5 Ke6 Bd8** and the pawn on d7 prevents its king from attacking the bishop). If however **3...f5** then **4 d7** is playable. Black can only reply **4...Ke7**, and **5 d8Q+ Kxd8** gives **3.4b**.

Now **6 Kc4**, gaining a tempo by an attack on the bishop as in **3.3**? No, Black can play **6...Bh8** and **7-8 Kf6**, and his pawn is safely defended. First must come **6 Ke6** pushing the pawn out of reach of its king, and after **6...f4** we do indeed have a finish like that of **3.3**: **7 Kd5 f3** (nothing better) **8 Kc4 B~ 9 Kd3** etc.

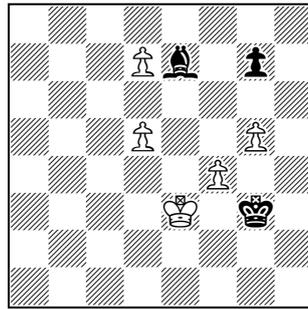
The play here is somewhat richer than that of **3.3**, but the richness has been achieved at some cost in complexity, and while it is a fine study my personal taste is more for the classic simplicity of **3.3**.

3.5 (M 16)



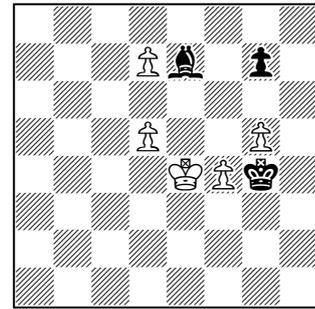
White to play and win

3.5a



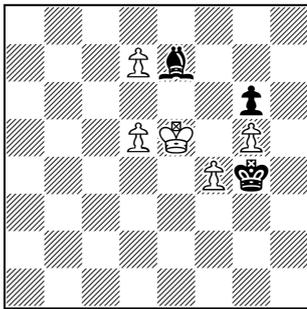
Reciprocal zugzwang

3.5b



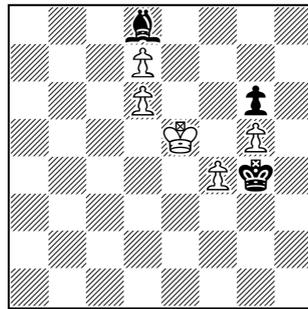
Reciprocal zugzwang

3.5c



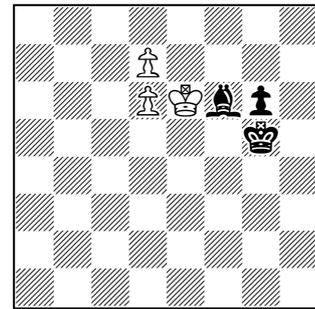
Reciprocal zugzwang

3.5d



Reciprocal zugzwang

3.5e



Reciprocal zugzwang

3.5 (*Basler Nachrichten*, 1929, correction) is Réti's most difficult study on the theme of bishop against pawns. White's basic objective is to reach **3.5e** with Black to play, but there will be several other positions of reciprocal zugzwang along the way. As with **1.4** and **2.2**, therefore, it is simplest to start at the end and work backwards.

From **3.5e**, Black to play, 1...Bd8 2 Kf7 Kf5 3 Ke8 B~ 4 d8Q Bxd8 5 Kxd8 g5 6 Ke7 and White will promote while Black's pawn is still at g3. White to play, however, 1 Kf7 Kf5 2 Ke8 Ke6, and White doesn't just fail to win, he loses.

From **3.5d**, Black to play, 1...Ba5 (1...Kh5 2 Ke6 and wins quickly) 2 f5 Kxg5 (2...gxf5 3 g6 etc) 3 f6 Bxc3+ (if instead 3...Kh6 then 4 Ke6 Bd8 5 Kf7 g5 6 Ke8 Bxf6 7 d8Q Bxd8 8 Kxd8 g4 9 Ke7 g3 10-11 d8Q g1Q 12 Qh8+ K~ 13 Qg8+ and wins Black's queen) 4 Kd5 (not 4 Ke6, when 4...Bxf6 gives **3.5e** with White to play) Bxf6 5 Ke6 and we have **3.5e**. White to play, 1 f5 Kxg5 2 f6 Kh6 3 Ke6, and because Black does not have to spend a move bringing his bishop back to d8 he is a tempo ahead of the line 3...Kh6 4 Ke6 above: 3...Kh7 4 Kf7 g5 5 Ke8 Bxf6 6 d8Q Bxd8 7 Kxd8 g4 8 Ke7 g3 9-10 d8Q g1Q, and Black draws because the check on h8 is not available.

From **3.5c**, Black to play, 1...Bd8 2 d6 and we have **3.5d**, or 1...Kh5 2 Ke6 Bd8 3 Kf7 etc. White to play, 1 Ke6 Bd8 2 Ke5 (else 2...Kxf4) Be7 and we are back at **3.5c**, or 1 d6 Bd8 and we have **3.5d**, or 1 Ke4 Bd8 2 d6 Ba5 3 Ke5 Bd8 and again **3.5d**.

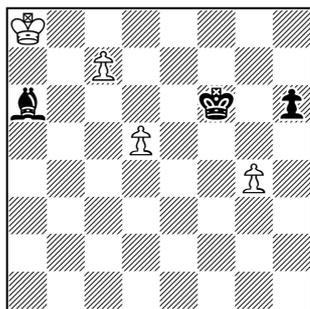
From **3.5b**, Black to play, 1...g6 2 Ke5 and we have **3.5c**, or 1...Bd8 2 Ke5 g6 (if 2...Kf3/Kg3 then 3 g6 Bf6+ 4 Kd6 wins quickly, and if anything else then 3 Ke6 does) 3 d6 and we have **3.5d**. White to play, 1 Ke5 g6 and again **3.5c**.

3.5a is more complicated, because we must take account of ...Bd8 and ...g6 as well as of king moves, but White has hopes of penetrating by Ke5, Ke6, Kf7 and of fixing the pawn on g7 by playing g6, and if Black plays ...g6 himself he speeds White's progress towards **3.5c** and **3.5d**. The given main line, Black to play, is 1...Kh4 2 Kd4 (threatening 3 Ke5 with a winning penetration) Kg4 3 Ke4 and we have **3.5b**, and variations given after 2 Kd4 are 2...Bd8 3 g6 Kh5 4 f5 Bf6+ (or 4...Kg5 5 Ke5 Bf6+ 6 Kd6 winning) 5 Kc5 Kg5 6 Kd6 Bd8 7 Ke6 Bf6 8 Kf7 Kxf5 9 Ke8 Ke5 10 d8Q Bxd8 11 Kxd8 Kxd5 12 Ke7 winning and 2...g6 3 d6 Bd8 4 Kd5 Kg4 5 Ke6 again giving **3.5c**. White to play is easy: 1 Ke4 Kg4 and we have **3.5b**, or 1 d6 Bd8 2 Ke4 Kg4 3 Ke5 g6 and we have **3.5c**.

And from **3.5** itself we can play 1 Kd3 threatening 2 Kd4 etc, with 1...Kg3 2 Ke3 giving **3.5a** and 1...Kg4 2 Ke4 giving **3.5b**.

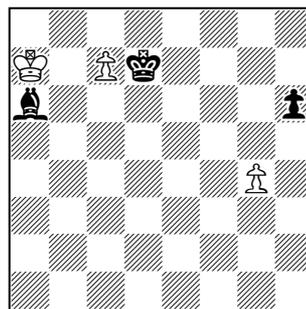
So the main line unfolds **1 Kd3 Kg3 2 Ke3** (see **3.5a**) **Kh4 3 Kd4 Kg4** (for 3...Bd8 and 3...g6 see the variations 2...Bd8 and 2...g6 in the analysis of **3.5a**) **4 Ke4** (see **3.5b**) **Bd8 5 Ke5 g6 6 d6** (see **3.5d**) **Ba5 7 f5 Kxg5 8 f6 Bc3+ 9 Kd5 Bxf6 10 Ke6** (see **3.5e**) **Bd8 11 Kf7 Kf5 12 Ke8** with a win; and what the good burghers of Basel made of it all, I dread to think.

3.6 (M 17)



White to play and hold the draw

3.6a

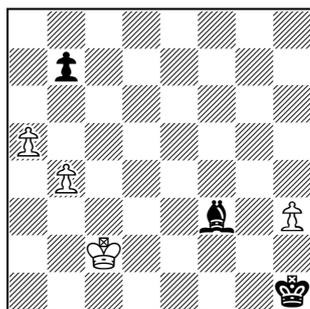


After 3 Ka7

In **3.6** (*Ostrauer Morgenzeitung*, 19 May 1929) an immediate 1 Ka7 is soon seen to fail (1...Bc8 2 Kb8 Bxg4 3 d6 Ke5 4 c8Q Bxc8 5 Kxc8 Kxd6 and Black's pawn will promote), and although 1 Kb8 gains a tempo on this it is not sufficient (1...Ke5 2 c8Q Bxc8 3 Kxc8 Kxd5 and 4-6...Kxg4, after which White's king will have no better seventh move than Kg6 or Ke4 and Black will shepherd his pawn safely home). The drawing line is **1 d6 Ke6** (1...Bc8 loses) **2 d7**, forcing Black's king to play **2...Kxd7** and block the diagonal c8-g4. Now and only now can White play **3 Ka7**, and we have **3.6a**.

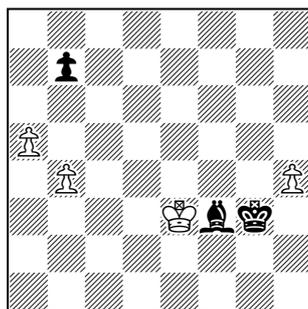
If Black tries 3...Be2 there follows 4 Kb8 Ba6 5 Ka7 repeating the position, and the same happens after 3...Bc8 4 Kb8 Ba6 5 Ka7. This leaves only **3...Kxc7**, but now White can play **4 Kxa6** and get back to the drawing square f1: **4...Kd6 5-7 Kd3 Kxg4 8 Ke2 Kg3 9 Kf1**.

3.7 (M 11 corrected)



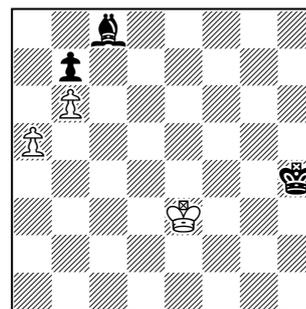
White to play and win

3.7a



After 3 Ke3

3.7b



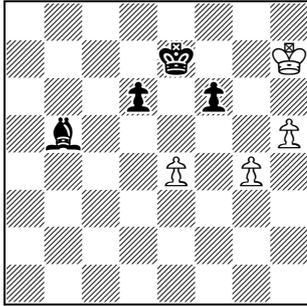
After 5...Bc8

In **3.7** (M 11, Commendation, *Kagan's Neueste Schachnachrichten*, 1922, correction by Benko), try 1 Kd2: no, 1...Bc6 2 h4 Be8, and although Benko's analysis stops there (at least as reported in *EG* 182) my computer agrees that Black can hold the draw. We'll look at 1 Kd3 in a moment, but the move to win is **1 h4**, and only after **1...Kg2** does White play **2 Kd2**. Not 2 Kd3, when Benko gives 2...Kf2 3 b5 Be2+, and this also deals with 1 Kd3 (1...Kg2, and if 2 h4 then 2...Kf2 transposing). Black continues **2...Kg3**, and **3 Ke3** gives **3.7a**.

Black's only hope now is to get his bishop back to c8, hence **3...Bg4**, but after **4 b5 Kxh4 5 b6 Bc8** all is seen to be in vain (see **3.7b**): **6 Kf4**, and all White has to do is to walk his king up to c7.

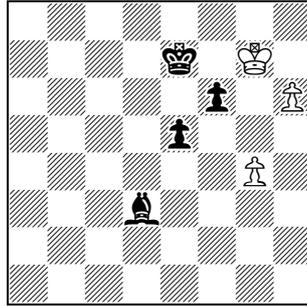
This was a companion piece to **3.1**. Réti had the kings on c5 and e2, intending 1 Kd4 Kf2 2 h4 Kg3 3 Ke3 and again we have **3.7a**, but Chéron and Bondarenko independently showed that Black could hold the draw by 2...Be2. Benko's modification (*Chess Life*, February 2008, quoted in *EG* 182) prevents this and even adds a bit in the need to play h4 before Kd2 and to avoid Kd3, but even so the study seems so far short of the subtlety of **3.1** (in particular, there appears to be nothing corresponding to the reciprocal zugzwang try 1 Kf4? Be2, 1 Kf5! Be2 2 Kf4) that I almost regret that it has not been allowed to remain unrescued among the casualties.

3.8 (M 15 corrected)



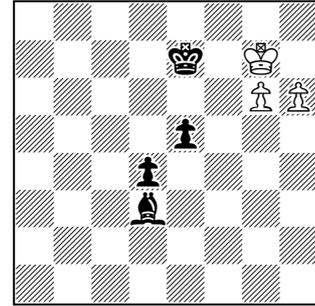
White to play and hold the draw

3.8a



Main line, after 3 h6

3.8b



2...fxe5, after 5 g6

The main line in **3.8** (M 15, 1st Honourable Mention, *Shakhmatny Listok*, 1928/I, correction by Benko) starts **1 Kg7 Bd3 2 e5 dxe5 3 h6** (see **3.8a**), and we have a curious positional draw in which Black cannot advance his pawns without blocking the defensive line of his bishop and letting the h-pawn through (the actual solution plays out a few moves to demonstrate the point). If instead 2...fxe5 then White's two connected passed pawns are comfortably strong enough to draw: 3 h6 d5 4 g5 d4 5 g6 (see **3.8b**) Bxg6 6 Kxg6 d3 7-8 h8Q d1Q 9 Qxe5+.

This was originally published with everything one file to the left (1 Kf7 Bc3 2 d5 cxd5 3 g6 etc) and with no mention of 2...exd5. In 1955, Bondarevsky queried this, citing the line 2...exd5 3 g6 Bh8 4 Kg8 Bb2 5 Kf7 c5 6-7 f6 c3 8-9 g8Q c1Q after which Black is a bishop up and each side still has a pawn on the board. Moving everything one file right kills this by taking away Black's move 3...Bh8, but it also spoils the given reply to an immediate 1 e5. As originally set (**3.8** with everything one file left), 1 d5 is given as being met by 1...Ke7 2 dxc6 Bc7 3 g6 Bxf4 4 Kh7 Be5 5 Kg8 Ke8 6 Kh7 Ba1 followed by 7 Kh6 Bf6 or 7 Kg8 e5 each with a Black win, but with everything one file right as in **3.8** the definitive results now available with K + 2P v K + B + P give the position after 1 e5 Kf7 2 exd6 Bd7 3 h6 Bxg4 as a draw. However, Black can also meet 1 e5 by 1...fxe5, and after 2-5 g8Q e1Q or 2 Kg7 e4 3-5 h8Q e1Q we again have positions in which Black is a bishop up and each side still has a pawn on the board. So the Bondarevsky line which destroyed the original setting appears to also have knocked out an unwanted dual in the new one. Benko's report of his correction in *EG* 182 makes no mention of 1 e5, but I haven't seen *Chess Life* (February 2008) where it originally appeared.