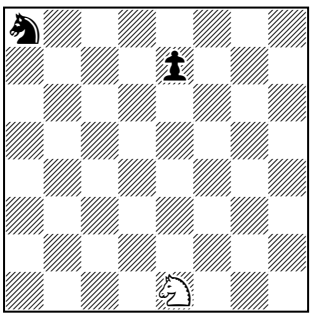
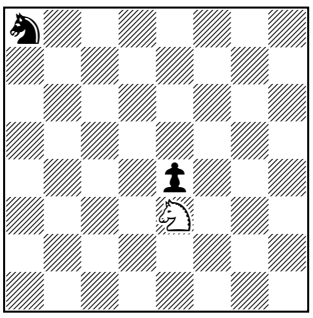
**A Losing Chess endgame re-analysed**

John Beasley, September 2018

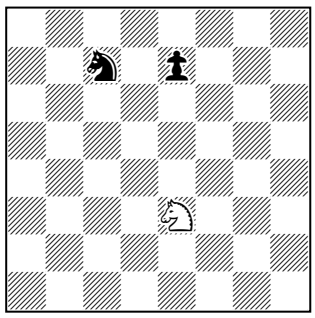
**1** below is an endgame study which I published in *Variant Chess* 30 in 1998. Unfortunately the computer has shown some details of the analysis given there to be faulty (though the overall conclusions were correct), and it seems to me that a more accurate presentation may be in order.



**1** - White to play and win



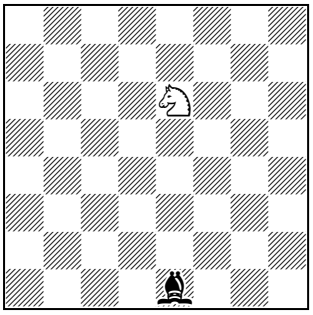
**2** - 1 Nc2 e6, after 4 Ne3



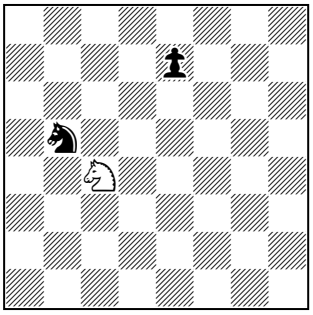
**3** - 1 Nc2 Nc7, after 2 Ne3

White has four moves. Let's start with **1 Nc2**. White can now deal with 1...e6 in short order (2 Nb4 e5 3 Nc2 e4 4 Ne3 gives **2**, after which the Black knight must emerge and allow a giveaway on d5) and 1...e5 in even shorter order (2 Nd4), which leaves 1...Nc7 and 1...Nb6. Let's look at **1...Nc7** first.

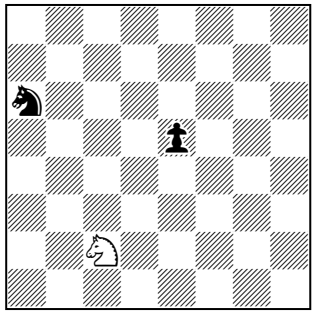
White can answer this by **2 Ne3**, giving **3**. He threatens a giveaway on d5, and if Black gives his own knight away first, 2...Nd5, White plays 3 Nxd5 with a standard win with a knight against a centre pawn: 3...e6 4 Nf6 e5 5 Nd5 e4 6 Nf4 e3 7 Nd3 e2 8 Nf4! e1B (other promotions lose at once) 9 Ne6, see **4**, and the bishop has no good move.



**4** - 2...Nd5 3 Nxd5, after 9 Ne6



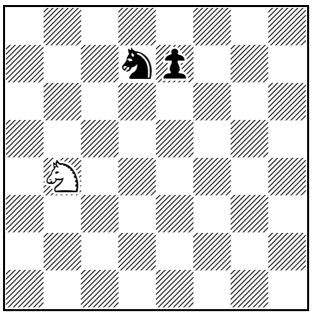
**5** - 2...Nb5, after 3 Nc4



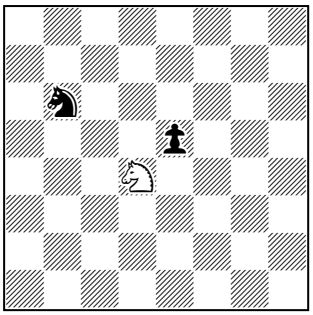
**6** - 2...Na6, after 5 Nc2

So the Black knight must play somewhere else from **3**. Try 2...Ne8: no, 3 Ng4 threatening a giveaway on f6, and if 3...e5 then 4 Nxe5 and White will win the N v N ending (we'll come back to this in a minute). Try 2...Ne6: no, 3 Ng4, and this time the giveaway on f6 is unavoidable. Try 2...Nb5: no, 3 Nc4 threatening a giveaway on d6, see **5**, with an N v N win after 3...e5 4 Nxe5 and an N v P win after 3...Na3 4 Nxa3 (4...e5 5 Nb5 e4 6 Nd4 with a left-to-right reflection of the play which led to **4**, or 4...e6 5 Nb5 e5 6 Nd4). Try 2...Na6: no, 3 Ng4 e6 (3...e5 4 Nxe5 and wins with N v N) 4 Ne3 e5 5 Nc2 (threatening both Nd4 and Nb4, see **6**) Nb4 6 Nxb4 e4 7 Nd3. Try 2...Na8: no, 3 Ng4 e6 4 Ne3 e5 5 Nc2 e4 6 Ne3, and we are back at **2**. So **1 Nc2 Nc7 2 Ne3** is a win for White.

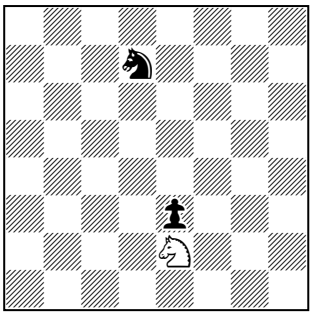
These N v N finishes will occur many times, so let us summarize them. N v N is a win for whoever is to move when the knights are on squares of the same colour. It follows that if the White knight can capture the Black pawn when it is on a square of *opposite* colour to that of the Black knight, White will win the N v N ending, but if White has to capture the pawn when it is on a square of the same colour as that of the Black knight then it is Black who will win.



**7** - after 1 Nc2 Nb6 2 Nb4 Nd7



**8** - after 2 Nd4 e5 instead

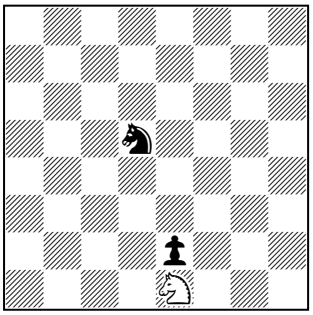


**9** - further after 5 Ne2 Nd7

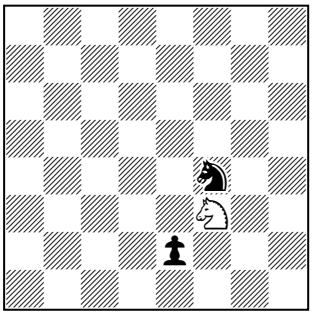
This leaves **1 Nc2 Nb6**. By analogy with 1...Nc7 2 Ne3, try **2 Nb4**, threatening another giveaway on d5 with an N v P win after 2...Nd5 3 Nxd5; yes, but Black has **2...Nd7**, see **7**, with a quick win in all lines (3 Na6/Nd3 Nc5 4 Nxc5 e6, 3 Nc6/Nd5 Nf8 4 Nxe7 Ng6, 3 Na2 Nc5 with 4 Nb4/Nc1 Nd3 5 Nxd3 e5 or 4 Nc3 Ne4 5 Nxe4 e5 6 N-- e4, 3 Nc2 Ne5 4 Na1 Nc6 5 Nb3/Nc2 Nd4).

All right, try **2 Nd4**. Now Black can play **2...e5** giving **8**, after which the play becomes complicated. From **8**, White must move to a light square, Black plays 3...e4, and White must move to a dark square again. If this square is d6, Black plays 4...Nd5, and we have 5 Nxe4 Nf6. If it is another square from which the knight attacks e4, Black plays 4...Nc8 followed by 5 Nxe4 Nd6. If it is a square from which the knight attacks d5 or c4 without attacking e4, Black plays 4...Nd5/Nc4 5 NxN e3. If it is a square two knight moves away from b6 from which the knight does not attack e4, d5, or c4 (it is soon seen that the only possibilities are b8, f8, and a7), Black gives his knight away on d7 or c8, promotes his pawn to a new knight (the White knight is too far away to prevent this), and wins with N v N.

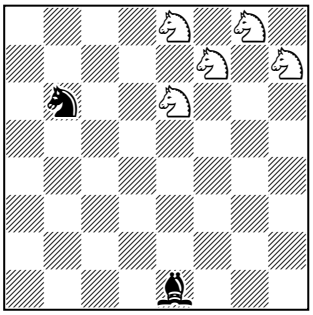
If instead White's fourth move is to a square four moves away from b6, Black plays 4...e3, and White must move to a light square three or five moves away from b6. If this square is e2, Black plays 5...Nd7 giving **9**, and after 6 N-- e2 he wins with N v N. If White has moved anywhere else, Black plays 5...e2, and White must again move to a dark square. If this is a square from which he attacks e2, Black plays 6...Nd7 and wins with N v N. If it is two knight moves away from b6 without attacking e2, Black gives up his knight, promotes to a new knight, and wins with N v N. If it is e1, Black plays 6...Nd5, giving **10** (6...Nc4 wins more quickly, but we shall need **10** later on). Now, after 7 Nc2/Ng2, we have 7...Ne3 8 Nxe3 e1B with a bishop giveaway next move; after 7 Nd3, 7...Nb4 and the same; and after 7 Nf3, 7...Nf4 (see **11**), with 8 Nd4/Ng1 Ng6 9 Nxe2 Nf4, or 8 Ne5/Ne1 Nd3 and 9...e1, or 8 Nh4 Ng2 and 9...e1, or 8 Ng5 Nh3 9 Nxh3 e1N and a win with N v N, or 8 Nd2/Nh2 e1B followed by a bishop giveaway and another win with N v N. And if White's sixth move is to some other square four knight moves away from b6, Black promotes to a bishop, White again must move to a light square three or five moves away from b6, and the only candidates not allowing an immediate bishop giveaway are shown in **12** (if instead White tries say 7 Ng2, Black will play 7...Bh4 and win with N v N). But the respite is only temporary, since Black can play 7...Ba5 with a quick win in all lines. If the White knight moves to a square two moves away from b6, Black gives his knight away and then his bishop; if it moves to d8, Black has 8...Bb4 followed by two giveaways; if it moves to any other square, say d4, the bishop attacks it and gives itself away on the square the knight has just left, and Black wins with N v N.



**10** - after 6 Ne1 Nd5

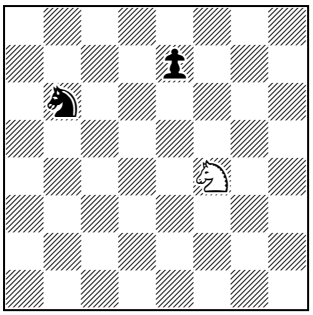


**11** - further after 7 Nf3 Nf4

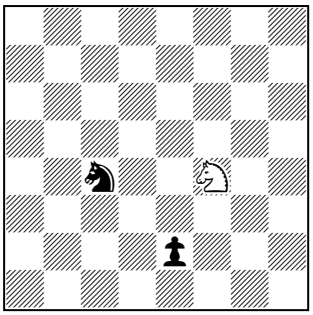


**12** - White's options after 6...e1B

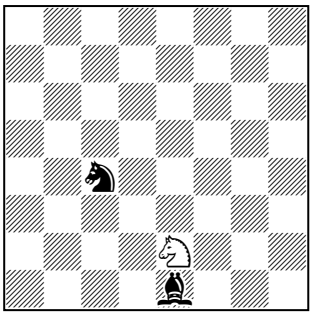
So, after 1 Nc2 Nb6, 2 Nb4 Nd7 and 2 Nd4 e5 are both wins for Black. Can White do better? Not by 2 Na3 or 2 Ne3, when 2...Nc4 3 Nxc4 e5 wins at once, nor by 2 Na1 or 2 Ne1, when Black can play 2...e5 with a similar win to that from **8**. So **1 Nc2 Nb6** is a win for Black.



**13** - after 1 Nd3 Nb6 2 Nf4



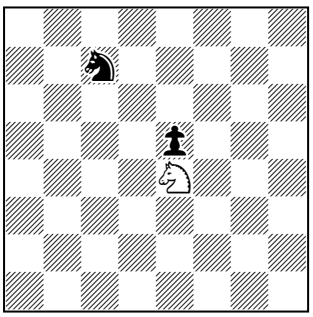
**14** - 2...Nc4, after 8 Nf4



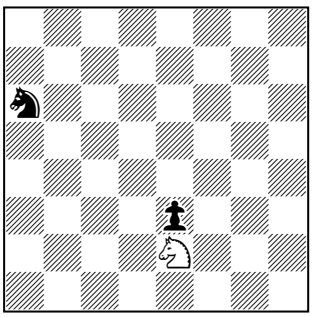
**15** - further after 8...e1B 9 Ne2

All right, let's try **1 Nd3**. **1...Nb6** can now be met by **2 Nf4** giving **13**, and this is soon seen to be good for White. He threatens a giveaway on d5, and 2...e6 3 Nxe6 gives him a win with N v N. So the Black knight must move. 2...Nd7 allows 3 Nh5 with a giveaway on f6 and 2...Nd5 3 Nxd5 gives a now familiar win with N v P, but what about 2...Nc4? White plays 3 Nh5 e6 4 Nf6 e5 5 Nh5 e4 6 Nf4 e3 7 Nh3 e2 8 Nf4 giving **14**, and if 8...N-- then 9 Nxe2 and he wins with N v N. Hence 8...e1B, but 9 Ne2 gives **15**, after which White threatens giveaways on c3 and g3, and if the bishop gives itself away first then again White wins with N v N.

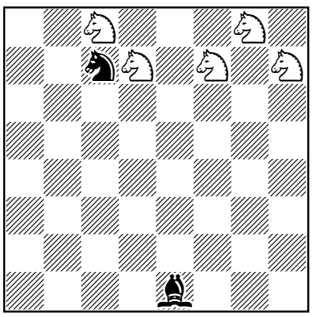
There remain 2...Na4/Na8/Nc8, but all these yield similarly: White can play 3 Nh5 etc, and eventually come down to **9** with the Black knight on a4, a8, or c8. All these positions are won for White, so **1 Nd3 Nb6 2 Nf4** is a win for White.



**16** - after 1 Nd3 Nc7 2 Nf2 e5 3 Ne4



**17** - after 5 Ne2 Na6

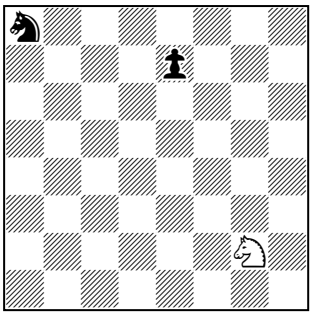


**18** - White's options after 6...e1B

White also wins after 1...e6 (2 Nb4 transposes into the line after 1 Nc2 e6) and 1...e5 (2 Nxe5 and wins with N v N), which leaves **1...Nc7**. Now 2 Nb4 loses to 2...Ne6 3 Na2 Nc5 transposing into the line after 1 Nc2 Nb6 2 Nb4 Nd7 3 Na2, 2 Nc5/Nf4 allow 2...Ne6 winning at once, 2 Ne5 is met by 2...e6 3 N-- e5 winning with N v N, and 2 Nb2/Nf2/Nc1/Ne1 can all be answered by 2...e5. The analysis of these last cases is similar to that of 1 Nc2 Nb6 2 Nd4 e5, but some of the details are different so let's see it through.

At move 3, White must move to a light square. If this is e4, we have **16**, to which the reply is elegant: 3...Na8 (only move) 4 N-- e4 and wins with N v N. Otherwise, Black plays 3...e4, and White must move to a dark square. If this square is c5, Black plays 4...Nd5 for 5 Nxe4 Nf6. If it is another square from which the knight attacks e4, Black plays 4...Na6 for 5 Nxe4 Nc5. If it is a square from which the knight attacks d5 without attacking e4, Black plays 4...Nd5 5 NxN e3. If it is a square two knight moves away from c7 from which the knight does not attack e4 or d5 (it is soon seen that the only possibilities reachable from b2/f2/c1/e1 are d4 and a3), Black gives his knight away on b5, promotes his pawn to a new knight, and wins with N v N.

If instead White's fourth move is to a square four moves away from c7, Black plays 4...e3, and White must move to a light square three or five moves away from c7. If this square is e2, Black plays 5...Na6 giving **17**, and after 6 N-- e2 he wins with N v N. If White has moved anywhere else, Black plays 5...e2, and White must again move to a dark square. If this is a square from which he attacks e2, Black can play 6...Na6 and win with N v N. If it is two knight moves away from c7 without attacking e2, Black gives up his knight, promotes to a new knight, and wins with N v N. If it is e1, Black plays 6...Nd5 repeating **10**. If it is any other square four knight moves away from c7, Black promotes to a bishop, White must move to a light square three or five moves away from c7, and the only candidates not allowing an immediate bishop giveaway are shown in **18**. But Black waits by 7...Bd2 (after Nc8/Nd7) or 7...Bf2 (otherwise), with a quick win in all lines. If the White knight moves to a square two moves away from the Black, Black gives his knight away and then his bishop; if it moves to any other square, the bishop attacks it and gives itself away on the square the knight has just left, and Black wins with N v N.



**19** - after 1 Ng2

So **1 Nd3 Nc7** is a win for Black. But 1 Nc2 Nc7 2 Ne3 and 1 Nd3 Nb6 2 Nf4 are wins for White, which suggests that **1 Ng2!** will be a win for White since it will allow **1...Nc7** to be met by **2 Ne3** and **1...Nb6** by **2 Nf4** (see **19**). This is indeed the case, since Black's other moves can be quickly mopped up (1...e6 2 Ne3 etc, 1...e5 2 Nf4).

To complete the solution, it is necessary to show that, after 1 Ng2, 1...Nc7 can be met only by 2 Ne3 and 1...Nb6 only by 2 Nf4, and also that 1 Nf3 does not work (it can in fact be met by either 1...Nc7 or 1...Nb6). However, these analyses involve nothing new, and we leave them as exercises for the reader's computer.

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The analyses above occasionally give preference to simplicity of exposition over distance to final capture. After 1 Nc2 Nc7 2 Ne3 Nb5, 3 Nc4 is the simplest and most natural move, but White can win slightly more quickly by playing 2 Ng4 (3...e6 4 Ne3 e5 5 Nc2, and if 5...Na3 then 6 Nxa3 e4 7 Nb1 e3 8 Nd2). After 1 Nd3 Nb6 2 Nf4, it is natural to mop up 2...Na4/Na8/Nc8 by repeating the line which we have just used to defeat 2...Nc4, but in fact there is a quicker win in every case. In the refutation line 1 Nc2 Nb6, it is simplest to meet 2 Na1/Ne1 by the move 2...e5 which we have just used to defeat 2 Nd4 (the analysis clearly carries across), even though 2...Nc4 provides a quicker win in each case. In the refutation line 1 Nd3 Nc7, it is simplest to lump all of 2 Nb2/Nf2/Nc1/Ne1 together and play the move 2...e5 which is needed to defeat 2 Nb2 and 2 Nf2, even though 2 Nc1 and 2 Ne1 allow quicker wins by 2...Ne6.

However, I do not think the treatment of the refutation lines 1 Nc2 Nb6 2 Nd4 e5 and 1 Nd3 Nc7 2 Nb2/Nf2 e5 can by significantly shortened. After 2...e5, Black's pawn is on a square of the same colour as that of White's knight, so he will simply leave his knight on c7 or b6 and push his pawn forward until White's knight blocks it. There are several possibilities, each of which must be examined: Black pawn on e5 blocked by White knight on e4 (diagram **16**), pawn on e4 blocked by knight on e3 (mopped up straight away by 4...Nd5 5 Nxd5 e3), pawn on e3 blocked by knight on e2 (diagrams **10**/**17**), pawn on e2 blocked by knight on e1 (diagram **11**), and pawn promoted on e1 (diagrams **12**/**18**). And while this procedure excludes any possibility of White's knight giving itself away to Black's pawn, it can threaten to give itself away to Black's knight, and to prove such a giveaway impossible appears to require a detailed case-by-case analysis. So I do not see how the treatment given here can be significantly shortened, though for readers already familiar with Losing Chess endings the amount of mopping-up detail can obviously be reduced.