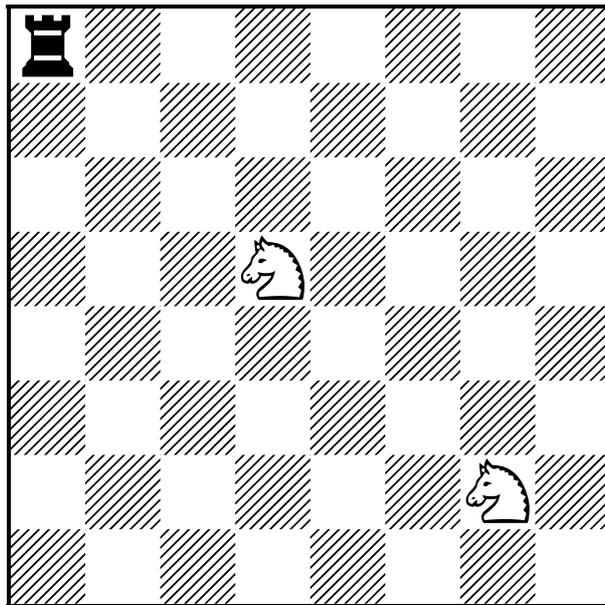


John Beasley

Three-man pawnless endings in Losing Chess



This is a corrected version of a document which has been circulating privately since January 1998 (originally under the title "Three-piece endings in Losing Chess"). Included therein was an invitation to anybody who had already performed this work himself to inform me, together with a statement that if I received no such notice within a year I would consider myself entitled to publish the document formally. I have indeed received no such notice, and I am therefore formally publishing the document and lodging the legal deposit copies as required by British law. Anybody already holding the version of July 1998 is advised that the present document differs only in the addition of this front page; anybody holding the original document of January 1998 should discard it.

7 St James Road, Harpenden, Herts AL5 4NX, England

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Three-man pawnless endings in Losing Chess

John Beasley, 7 St James Road, Harpenden, Herts AL5 4NX, England; July 1998

In January 1998, I produced an informal document entitled “Three-piece endings in Losing Chess” in which I examined all the three-man pawnless endings in Losing Chess with the aid of a computer-generated database. It was my intention at the time to wait for a year to see if anyone would claim to have done this work already (I was aware of the work by Gyorgy Evseev on two knights against one, quoted in the *British Chess Magazine* in November 1992 and subsequently described in an article “Finales de cavaliers en ‘qui perd gagne’ ” by Evseev and Christian Poisson in *Rex Multiplex* in April 1993, but not of any other), and then to publish it more formally. However, nobody has made such a claim so far, and if anyone else had discovered the remarkable lose-a-move win with two knights against rook, where a player moving only a knight can contrive to transfer the move to his opponent, I think he would have done as I have done and published it everywhere in sight. I therefore presume, a little to my surprise, that this work had *not* been done previously by computer (though a number of endings had been examined by hand, notably by Fabrice Liardet in an article in *Schweizer Schach-Magazin* in August-September 1991), and the existence of a few unfortunate errors in the original document makes it desirable that a corrected version be issued.

This is therefore a revised version of the original document. I have retitled it to clarify its nature, and I hope I have removed all errors. The most serious of these was a mis-statement of a position of reciprocal zugzwang. To avoid double counting, the computer normalizes all positions, and its normalization is not always the most convenient for subsequent discussion. Some positions have therefore been renormalized, and one (in the ending R v K+B) was originally renormalized wrongly. I hope there is no such error in the present document, but as an insurance I have added appendices giving the database statistics and reciprocal zugzwangs as actually generated by the computer. Everything displayed in the fixed-pitch “Courier” font has been copied from computer output with no change beyond formatting into columns, but positions in the proportionally-spaced font used for the bulk of the text may have been renormalized and should be checked against Appendix 2 if there is any doubt.

I have no authority to waive the rights of others, but in so far as anything in this document is original with me I am happy that it be freely copied or quoted. I ask only that there be appropriate acknowledgement. If it appears that any of this work has been performed previously, other than as acknowledged in the text, or if any error exists in the present document, I shall be glad if the matter is brought to my notice.

My thanks to Fabrice Liardet for reporting the errors in the original document, and for other valuable comments. Since the original document was written, Laurent Bartholdi has generated a database covering all three-man positions with and without pawns, but I have not extended the present document beyond pawnless positions.

Preliminary note 1: “trivial” endings

Many endings in Losing Chess are “trivial”: the player to move is forced to make an immediate capture, or can win by an immediate sacrifice. These trivial endings distort raw computer statistics to no useful purpose, and we shall always disregard them. The one-against-one draw with bishops of opposite colours is also trivial.

In the original document, an ending was also normally disregarded if a player could force his opponent to make a disadvantageous capture on his first or second move (a so-called “short” ending), but I now consider this a mistake and in the present document a short ending is treated like any other non-trivial ending.

Preliminary note 2: two-man pawnless endings

Although these are well known, it is convenient to list them here, partly for completeness and partly because they provide a gentle introduction to some ideas that we shall use later.

Endings in which each side has a line-moving piece (queen, rook, or bishop) are normally trivial, but there are exceptions in which a player can attack his opponent without allowing him to sacrifice in return. A queen or rook can do this against a bishop (for example, Q/Rb1 against Bb8) and a bishop against a rook (Bb1 against Rd3), but a queen cannot do it against a rook. All these are wins for the “attacking” side, because his opponent must move away and the attacker then sacrifices on the square which he has just been attacking.

Queen, rook, or bishop against knight are normally wins, but there are a couple of wins by “attack and wait” for knight against rook (typically, Nb1 against Rd2) and one for knight against bishop (Na2 against Bc1). Here and elsewhere, we count positions which can be rotated or reflected into each other as the same. There is also a win by “domination” for knight against bishop (Ne6 against Be1). The knight does not threaten anything in this position, but the bishop has no safe move and will have to allow the knight to sacrifice itself next move.

Knight against knight is a win for whoever is to move when the knights are on squares of the same colour. There are no exceptions, not even trivial ones.

Queen or rook against king are wins. **Bishop against king** is normally a draw, but there are three positions in which the bishop wins by domination (Ba4 against Ka1, Bc4 against Kc1, Bd4 against Kd1). Set Bb4 against Kb1, however, and the king can hold the draw by playing Ka1.

King against knight is normally a win, but there are some “attack and wait” wins for the knight against a king on the edge (typically, Nb2 against Kd1) and one “domination” win against a king in the corner (Nd4 against Ka1).

King against king is a draw.

The rest of this document will consider only the normal 8 x 8 board, but we may mention in passing that there are a few cases in which **the result may depend on the board size**. Qg3 to move against Kc1 wins only by playing to g8, h3, or h4, and on a 7 x 7 board this position is a win for the king. Be1 to move against Ne6 loses on 8 x 8 and 9 x 9, but wins on 10 x 10 and beyond. King against knight is a win on all boards up to and including 12 x 12, but the knight may be able to run indefinitely on a 13 x 13 or any larger board.

Preliminary note 3: castling

There is no three-man position in Losing Chess in which castling is the only move to win or to draw. We can therefore ignore castling altogether.

Preliminary note 4: move counts in computer databases

It is a property of computer databases that they normally count moves *up to a decisive change in material*, not to the final move of the game. Set wKe4 against bRa8 and bKc6, and the computer says that Black can win in one move by playing ...Kd5 since the capture KxK gives a lost position with K v R. In fact a White player who has nothing better to do with his time can drag out the lost K v R ending for a further 17 moves, but such behaviour is normally ignored. It is possible to construct databases so that they count moves up to the final move of the game and there are situations where this is desirable, but in the present circumstances it would tend to obscure what is going on rather than to clarify it.

We now proceed to three-piece endings. We always assume that White always has the single piece, and except where we are considering positions of reciprocal zugzwang we normally assume that it is the single piece to move.

1. Lone queen

Queen against queen and anything and **queen against rook and anything** are always trivial with the queen to move, as is **queen against two bishops**.

Queen against bishop and knight offers three non-trivial wins for the queen to move (Qg4 or Qh3 against Ba8/Nb7, Qh3 against Ba8/Nc6) and one non-trivial loss (Bb8/Nc7 against Qh4). In the latter position, whoever is to move loses.

Queen against two knights offers 56 non-trivial wins for the queen to move and 21 non-trivial losses. In two cases, the knights can hold out until move 3 (Qe1 against Na8/Nb8 or Nh7/Nb6). The latter case illustrates the computer’s method of counting, since it gives the optimal line of play as 1 Qe2 Na8; it regards 1...Nc4 as inferior because 2 QxN immediately gives a won two-man ending for the queen, even though the final move of the game (if Black plays it out to the bitter end) occurs no sooner.

There are 21 non-trivial positions in which the side to move loses, listed by the computer as follows:

Na8/Ne8 v Qh3	Nc8/Ne6 v Qh1	Nc7/Nd6 v Qg1
Na8/Nc7 v Qh4	Nc8/Nc4 v Qh1	Nc7/Nb4 v Qg1
	Nc8/Nc2 v Qh5	
Nb8/Nc8 v Qh5		Nd7/Ne6 v Qh1
Nb8/Nd6 v Qg1	Nd8/Ne7 v Qh2	Nd7/Nf5 v Qa2
Nb8/Nb4 v Qg1	Nd8/Nf6 v Qa3	Nd7/Nc4 v Qh1
Nb8/Nb2 v Qg5	Nd8/Nc5 v Qh2	
Nb8/Nc1 v Qh4	Nd8/Nf4 v Qa1	Nd6/Nf4 v Qa1
	Nd8/Nh4 v Qa1	

Fabrice Liardet points out that these are precisely the 21 non-trivial losses for the queen to move, and that they all have the property that the queen is immediately dominated.

Queen against king and bishop introduces our first drawn positions. There are two such, Qh4 against Kc6 and Ba8 or Bb7, and Paul Byway exploited them in a study published in 1995. Suppose bBa8. White’s only safe move is 1 Qe1; Black’s only safe reply is 1...Bb7, giving the other drawn position; White can only play 2 Qh4;

Black must reply 2...Ba8, and we are back where we started.

There are nine non-trivial wins for the queen to move, and nine non-trivial losses. In two of the latter, Qf2 against Bc8/Kd7 and Qg2 against Bd8/Ke7, the queen can hold out until move 3 (taking Qf2 against Bc8/Kd7, 1 Qg1 Ke6 etc).

There are three non-trivial positions in which the side to move loses: Bc8/Kd7 against Qf2, Bd8/Ke7 against Qg2, Be8/Kf7 against Qh2. Why not Bb8/Kc7 against Qe2? Because White to play can win by Qe1 or Qh5.

Queen against king and knight is a tricky ending. The computer counts 127 non-trivial wins for the queen to move, 172 non-trivial losses, and 19 draws; many positions require detailed analysis, but we may note that there are no non-trivial positions with the knight in the four central squares and that all non-trivial positions with the knight on one of the twelve squares surrounding the centre are lost for the queen. (We shall see in many endings that the player with a knight should normally get it as near to the centre as he safely can.) The longest win occurs with Qg5 against Na8/Kb8: 1 Qh5 Kb7 (best) 2 Qd1 Kb6 3 Qg4 Ka6 4 Qe6/Qg6 (a remarkable resource) and Black has an uneviable choice between moving his king to a defended square (4...Kb6), defending it (4...Nc7), shielding it by a defended knight (4...Nb6), or moving it away and allowing White to sacrifice on the square it has just left.

The longest loss occurs with Qa1 against Nf8/Kg4, when the computer plays

1	Qa1-b2 !	Kg4-h4 !	5	Qa1-b2 !	Kh4-g4 !
2	Qb2-a1 !	Kh4-g3 !	6	Qb2-a1 !	Kg4-f3 !
3	Qa1-c1 !	Kg3-h3 !	7	Qa1-a5 !	Kf3-g2 !
4	Qc1-a1 !	Kh3-h4 !			

with a sacrifice after White's next move. Here and elsewhere in computer output, "!" indicates a unique optimal move: other moves by the losing side lose more quickly, other moves by the winning side either delay the win or forfeit it altogether. The play from Black's 1st move to his 5th is a "lose-a-move" manoeuvre which returns to the same position with White to play.

The computer lists the following positions as drawn:

Na8/Kg8 v Qd1	Nb8/Ka4 v Qf2
Na8/Kf7 v Qd2	Nb8/Ka3 v Qg5
Na8/Kg7 v Qe2, d1, e1	Nb8/Kb3 v Qh5, g1
Na8/Kh7 v Qe2, d1, e1, f1	
Na8/Kf6 v Qd1	Nc8/Kh7 v Qf3
Na8/Kg6 v Qe1	Nc8/Kh6 v Qf1
Na8/Kh6 v Qe1, f1	

There are 13 non-trivial positions in which the side to move loses, listed by the computer as follows:

Nb8/Kd8 v Qf2	Nd8/Kc8 v Qa3	Nb7/Kf8 v Qh3
Nb8/Kb7 v Qg5	Nd8/Ke8 v Qg3	
	Nd8/Kf8 v Qh2	Nc7/Ka4 v Qf2
Nc8/Ke8 v Qg2	Nd8/Kb1 v Qg3	
Nc8/Ka5 v Qf3	Nd8/Kf1 v Qa3	Nc6/Kf8 v Qh3
Nc8/Ka3 v Qf1		

In addition, there are two positions in which the queen loses with the move but can draw without it (Nb8/Kc3 against Qh5, Nc8/Kh5 against Qf1), and five in which the same is true of the pieces (Na8/Kf7 against Qd2, Nb8/Ka4 against Qf2, Nb8/Ka3 against Qg5, Nb8/Kb3 against Qg1, Nc8/Kh7 against Qf3).

Some positions in this ending were exploited in 1995 by Paul Byway and in 1997 by Vincent Geerlings.

Queen against one king is a win for the queen, but the queen has limited freedom of action. The addition of a second king restricts her further, and **queen against two kings** is not easy to summarize. The computer counts 63 non-trivial wins for the queen to move, 121 non-trivial losses, and 105 draws. The list of draws is as follows:

Ka8/Kc8 v Qh6, h5	Ka8/Kg6 v Qe1
Ka8/Ke8 v Qc3, c2, c1	Ka8/Kh6 v Qc4, e1
Ka8/Kf8 v Qd3, h3, c2, d2, c1, d1	Ka8/Kh2 v Qf6, c4
Ka8/Kg8 v Qd2, e2, c1, d1, e1	Ka8/Kh1 v Qe6, f6
Ka8/Kh8 v Qd2, c1, d1	
Ka8/Kb7 v Qg5, h5, h4	Kb8/Kc8 v Qg6
Ka8/Ke7 v Qc2, c1	Kb8/Kf8 v Qd3, d2, d1
Ka8/Kf7 v Qd2, c1, d1	Kb8/Kg8 v Qd2, d1
Ka8/Kg7 v Qe2, d1, e1	Kb8/Kb7 v Qd2
Ka8/Kh7 v Qe2, d1, e1, f1	Kb8/Kf7 v Qd2, d1
Ka8/Ke6 v Qc1	Kb8/Kg7 v Qe2, d1, e1

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Kb8/Kh7 v Qe2, d1, e1, f1	Kc8/Kg6 v Qe1
Kb8/Kf6 v Qd1	Kc8/Kh6 v Qa2, f2, e1, f1
Kb8/Kg6 v Qe1	Kc8/Kb3 v Qh6, h5, g1
Kb8/Kh6 v Qe1, f1	Kc8/Kb2 v Qh5
Kb8/Ka4 v Qg6, h6	
Kb8/Ka3 v Qg6, h6, g5, h5, f1	Kd8/Ke8 v Qb2, a1, b1
Kb8/Kb3 v Qh6, g5, h5	Kd8/Kc7 v Qa1
Kb8/Kc3 v Qh5	Kd8/Ke7 v Qg1, h1
Kb8/Kb2 v Qh5, h4	
Kb8/Kc2 v Qh4	Kb7/Kf7 v Qd2, d1
Kb8/Kb1 v Qh5	Kb7/Kg7 v Qd1
Kb8/Kc1 v Qh4	Kb7/Kf6 v Qd1
Kb8/Kd1 v Qf6	Kb7/Kg6 v Qe1
Kc8/Kd8 v Qa2, f1, g1	Kc7/Kg6 v Qe1
Kc8/Kg7 v Qe2, e1	

Most are similar in character to the draws with queen against king and knight, the queen having only one or two safe moves at each stage, but the positions with adjacent or near-adjacent kings introduce a new motif. Consider Ka8/Kb7 against Qg5, which is one of the positions where the queen has the initiative. The queen would defeat either king on its own by staying just out of range and waiting for its opponent to retreat, but with two kings Black can set one forward as a bastion and play tempo moves with the other. So Black keeps his forward king at b7 and moves his other king carefully around the squares a8-b8-a7 (one square is always safe), and White can never approach.

The longest win for the kings occurs with Ka7/Ke6 against Qc1, when the computer plays

1 Qc1-c2 !	Ke6-e7 !	4 Qc2-c1	Ka6-a5 !
2 Qc2-c1 !	Ka7-a6 !	5 Qc1-h1 !	Ke8-f7 !
3 Qc1-c2 !	Ke7-e8 !	6 Qh1-c1	Kf7-e6 !

with a sacrifice after White's next move. White can also try 4 Qh2, but the sequel 4...Ka5 5 Qg2 Kb4 6 Qh2 Kc4 is no better.

The only long wins for the queen occur with Qh6 or Qh5 against Kb8/Kc8. White plays 1 Qg6; Black must reply 1...Ka8 to avoid immediate loss; and White plays 2 Qb1 with a sacrifice next move.

There are four non-trivial positions in which the side to move loses (Kc8/Kd8 against Qh6, Kc8/Ke1 against Qg6 or Qa3, Kd8/Ka5 against Qf3). There are also six positions in which the queen loses with the move but can draw without it (Ka8/Kd8 against Qh6, Kb8/Ka4 against Qf6, Kb8/Kh2 against Qf6, Kc8/Kd8 against Qa1, Kd8/Kc7 against Qa2, Kd8/Ke7 against Qg2) and three in which the same is true of the kings (Ka8/Kh6 against Qc4, Kb8/Kc8 against Qg6, Kb8/Kd1 against Qf6).

2. Lone rook

Rook against queen and anything and rook against rook and anything are always trivial with the rook to move.

Rook against two bishops, with the rook to move, is always trivial with unlike bishops, but with like bishops it offers 32 non-trivial wins for the rook and 8 non-trivial losses. These are typified by Rd5 against Ba8/Bb7 (the rook wins) and Rd5 against Ba8/Bg8 (it loses). With the bishops to move, Fabrice Liardet points out some short but interesting non-trivial losses, for example Ba5/Bd1 against Rh8. Here, a bishop must sacrifice itself immediately if Black is to stay alive, but the only two sacrifices (...Bd8 and ...Bh5) each lead to exceptional positions where B v R loses even though the bishop has the move.

Rook against bishop and knight offers 321 non-trivial wins for the rook to move, 14 losses, and 24 draws. Twelve of the draws are given by bNd6, bB on b8 or c7, and wR somewhere in the rectangle g3-h3-h1-g1, where it is easily seen that if bB moves off the line b8-c7 or wR moves outside the rectangle g3-h1 the other side can sacrifice immediately. Ten of the other positions are similar (Rh2 or Rh1 against Bb8/Ne5, the same against Bd6/Ne5, Rg2/h2h1 against Bb7/Nd5, and the same against Bc6/Nd5). In the other two positions, we have Rh2 against Bb8/Nc5 or Bc7/Nc5. Here White plays 1 Rh1; if Black sacrifices by 1...Bh2, he gives himself a lost N v R ending, so he must hide his bishop by 1...Ba7 or 1...Bb6; White plays 2 Rg1, and we have a reflection of one of the positions already described. The positions with Rh1/h2 against Bb8/d6 and Ne5 were exploited in a study published by R. Sekhar and R. Shankar in 1987. Note that Black cannot play ...Bc7 in these positions because White can reply Ra2; the sacrifice ...Ba5 gives White a winning ending with R v N.

The non-trivial wins for the rook also offer points of interest. The longest has Rh3 against Bc8/Nc6, and White starts 1 Rh1/Rh2. Black will lose if he sacrifices on h3, so he hides his bishop by 1...Bb7. White now plays

2 Rg1/Rg2, and Black must play 2...Ba8 to keep his bishop hidden. But this restricts the bishop to the long diagonal, and now White can attack the knight, 3 Rg6 say, and Black cannot leave it without defence.

The non-trivial losses for the rook are typified by Rh1 against Ba8/Ne5. The rook must move away, and while a sacrifice on h1 would lose Black always has a winning alternative sacrifice. There is also a loss by domination (Rh1 against Bb7/Ne4). Note however that the apparently similar position with Rh1 against Ba8/Ne4 is a win for the rook. White can play 1 Rh7/Rb1, and the only sacrifice available to Black is a losing one; alternatively, he can attack the knight, and Black cannot leave it without defence.

Rook against two knights is one of the most interesting endings. The computer counts 641 non-trivial wins for the rook to move, 22 non-trivial losses, and 197 draws.

To get a feel for the material, let us set the rook at h1, one of the knights in an oppressive position at e4, and the other knight in the top left -hand corner.

- With bN on a8, White soon wins. He plays 1 Rh2, and Black must retreat since 1...Nd2/Nf2 2 RxN would be a win for the rook against the remaining knight. Black's two sensible moves (1...Nc5 and 1...Nd6) are of equal value; let us choose 1...Nd6. White plays 2 Re2, and bN must retreat again because even 2...Ne8 3 RxN would give the win to White. Black's best move is 2...Nb5; White plays 3 Re4 forcing Black to retreat for the third time; and we have 3...Na7 4 Rd4 with a sacrifice next move.
- With bN on b8, the game is drawn. White can play 1 Rg1 forcing Black to retreat, but after 1...Nc5 he has nothing better than 2 Rf1/Rg2. Black can now hold the draw most simply by keeping one knight at c5 and playing the other back and forth between c6 and b8. This restricts wR to the rectangle f2-h2-h1-f1, and progress is clearly impossible.
- With bN on b7, wR is dominated and any move is immediately fatal.

If it is Black to move in this last position, he must lift the domination, but he can still win. The computer gives the following:

1	Nb7-c5	6	Rh8-h7 !	Nc3-b1 !
2	Rh1-h8 !	Nc5-b3 !	7	Rh7-h8 !	Nb1-d2 !
3	Rh8-h7 !	Nb3-d4 !	8	Rh8-h7 !	Nd2-b3 !
4	Rh7-h1 !	Nd4-b5 !	9	Rh7-h8 !	Nb3-c5 !
5	Rh1-h8 !	Nb5-c3 !	10	Rh8-h1 !	Nc5-b7 !

Apart from the opening move, where 1...Nd6 is equivalent by symmetry, each of these is a unique optimal move, and at the end we have returned to the original position with White to play. We can therefore add this to the small and delightful class of chess positions in which a player moving only a knight can contrive to transfer the move to his opponent.

The play rewards detailed study. At move 1, moves by the e-knight can be shown to lose, and 1...Nd8/Na5 allow White to hold the draw (after say 1...Nd8, White can play 2 Rg1, driving bNe4 from its commanding position). Play is now straightforward until after Black's third move, when we have the knights side by side on d4 and e4 and the rook on h7. If White now plays 4 Ra7, we have 4...Ng3 5 Ra8 Ngf5 6 Ra1 Ng7 echoing the original domination. If he plays 4 Rh8, we have 4...Nc3 5 Rh7 (5 Rg8 is equivalent by symmetry) Nce2 6 Ra7 (if 6 Rh8 then 6...Nb5 dominates wR at once) Ng3 and we have the same position as after 4 Ra7 Ng3. This leaves his actual move 4 Rh1, but even this allows Black to dictate the play (White's only significant choice occurs at move 8, when he can play Ra8 instead of Rh7, but then 8...Ng5 dominates wR at once). It may seem strange that Black has to play his knight right down to b1 at move 6, but nothing else wins. If at any time he puts his knights an even number of squares apart on the same rank or file, White can move to the line of bisection and win at once (for example, 4...Ne6? 5 Rh5), and this seriously limits his freedom of action.

The position with the knights side by side on d4 and e4 is thus seen to be a key winning position in this ending. The rook has only three pairs of temporarily safe squares, h8/a8, h7/a7, and h1/a1, and each loses in due course.

A complete list of non-trivial positions in which the rook to play loses is as follows:

Nb8/Ne5, Rh2 or h1;	Nd7/Ne5, Ra2 or h1;	Nd6/Ne4, Ra1 or h1;
	Nd7/Nc4, Rg1;	
Nb7/Ne4, Rh1;	Nd7/Ne4, Ra1 or h1;	Nd5/Ne5, Rh8, h2, or h1;
	Nd7/Ng4, Ra1;	Nd5/Ne4, Rh1.
Nc7/Ne6, Rh3;		
Nc7/Nd4, Rg1 or h1;	Nc6/Nd5, Rg2 or h1;	
	Nc6/Ne5, Rh2 or h1;	

We have already met several of these during the play above; in the case of the rest, Black can either play to a

position with his knights side by side in the centre and then reel off the win already shown, or play to join this solution after a few moves. With Nd5/Ne4 against Rh1, for example, we have 1 Rh8 (1 Ra1 is equivalent) Ndc3 2 Rh7 Nb1, and we are back on familiar territory.

If Black cannot establish a knight in the centre, it is White who has the winning chances, and the defensive strategy is to place one knight as a bastion as far from the corner as possible and play tempo moves with the other. The computer gives the complete list of draws as follows:

Nb8/Ne6 v Rh2, h1	Nc7/Nb6 v Rh1
Nb8/Nb5 v Re2, f2, g2, h2, e1, f1, g1, h1	Nc7/Nd6 v Rg3, h3, g2, h2, g1, h1
Nb8/Nc5 v Rf2, g2, h2, f1, g1, h1	Nc7/Nb5 v Rf2, g2, h2, f1, g1, h1
Nb8/Ne4 v Rh1	Nc7/Nd5 v Rg2, h2, g1, h1
	Nc7/Nb4 v Rf1, g1, h1
	Nc7/Nc4 v Rf1, g1, h1
	Nc7/Ne4 v Rh1
Nc8/Nd7 v Rg4, h4, g3, h3, g2, h2, g1, h1	
Nc8/Nd6 v Rg3, h3, g2, h2, g1, h1	Nd7/Ne7 v Ra4, a3, a2, a1
Nc8/Nb5 v Rf2, g1, h1	Nd7/Nc6 v Rg3, h3, g2, h2, g1, h1
Nc8/Nc5 v Rf2, g2, h2, f1, g1, h1	Nd7/Nd6 v Ra3, g3, h3, a2, g2, h2, a1, g1, h1
Nc8/Nd5 v Rg2, h2, g1, h1	Nd7/Ne6 v Ra3, h3, a2, h2, a1, h1
Nc8/Ne5 v Rh2, h1	Nd7/Nf6 v Ra3, a2, a1
	Nd7/Nb5 v Rg2, h2, h1
Nd8/Ne7 v Ra4, h3, h2, h1	Nd7/Nc5 v Rg2, h2, g1, h1
Nd8/Nc6 v Rg3, h2, h1	Nd7/Ne5 v Rh2, a1
Nd8/Ne6 v Ra3, h3, a2, h2, a1, h1	Nd7/Nf5 v Ra2, a1
Nd8/Na5 v Rg2, h1	Nd7/Ng5 v Ra2, a1
Nd8/Nc5 v Rg2, h2, g1, h1	Nd7/Nc4 v Rh1
Nd8/Nd5 v Ra2, g2, h2, a1, g1, h1	Nd7/Nd4 v Ra1, g1, h1
Nd8/Ne5 v Ra2, h2, a1, h1	Nd7/Nf4 v Ra1
Nd8/Nf5 v Ra2, a1	
Nd8/Ng5 v Ra2, a1	Nc6/Nd6 v Rg3, h3, g2, h2, g1, h1
Nd8/Nc4 v Rg1, h1	Nc6/Nd5 v Rh2
Nd8/Ne4 v Ra1, h1	Nc6/Ne4 v Rh1
Nd8/Ng4 v Ra1	Nd6/Ne6 v Ra3, a2, a1
Nd8/Nh4 v Ra1	Nd6/Nc5 v Rg2, h2, h1
	Nd6/Nd5 v Ra2, g2, h2, a1, g1, h1
Nb7/Ne7 v Rh4	Nd6/Ne5 v Ra2, h2, a1, h1
Nb7/Nc6 v Rf3, g2, h2, h1	Nd6/Nf5 v Ra2, a1
Nb7/Ne6 v Rh3, h2, h1	Nd6/Nf4 v Ra1
Nb7/Nd5 v Rh1	
Nb7/Ne5 v Rh1	Nd5/Ne4 v Rh8
Nc7/Nd7 v Rg4, h3, h2, h1	

The key positions are those in which the knights are side by side, and we see that d7/e7, d7/d6, c6/d6, d6/e6, and d6/d5 draw in all cases, and that c7/d7 draws against g4/h3/h2/h1 but not otherwise. In the last case, we might also expect draws with wR on h4 or g3-g1, but White can play 1 Rg4 and we have 1...Na8 (if Black gives ground by 1...Nb8 then 2 Rf4 wins easily) 2 Re4 Nb8 (now Black must give ground whether he likes it or not, because 2...Ne5 3 RxN is a win for the rook) 3 Re5. With wR on g4 or h1-h3, however, White has nothing better than 1 Rh4, and now Black can play 1...Ne8. White must play 2 Ra4, else 2...Nd6 will draw; Black plays 2...Ng7; White must retreat, say by 3 Ra3, and Black has gained enough ground to draw (3...Ne6 etc).

Two other positions are of interest: Nc6/Nd5 against Rh2, and Nd5/Ne4 against Rh8. Here the knights have gained a strong position, and it is the rook which is trying to hold the game. In the first case, Black threatens 1...Ne5 and 1...Nd4, in each case with the standard win, but White can hold him at bay by 1 Rg2 and now it is Black who has to be careful (only 1...Ne8 and the symmetrically equivalent 1...Na5 hold the draw). In the latter case, White would lose if his rook were on a8, as we have already seen, but with the rook on h8 White can play 1 Rh1 (or 1 Ra8) and Black can never achieve a winning position.

There is a single non-trivial position in which the side to play loses (Nc7/Ne6 against Rh3), but there are five positions in which the rook loses with the move but can draw without it (Nd7/Ne5 against Ra2, Nd7/Nc4 against Rg1, Nd7/Ng4 against Ra1, Nc6/Nd5 against Rg2, Nd5/Ne4 against Ra8) and seven in which the same is true of the pieces (Nc8/Nb5 against Rf2, Nd8/Ne7 against Ra4, Nd8/Nc6 against Rg3, Nd8/Na5 against Rg2, Nb7/Ne7 against Rh4, Nb7/Nc6 against Rf3, Nc7/Nd7 against Rg4).

It is a finely poised ending, apparently first examined by Fabrice Liardet in 1991.

Rook against king and bishop, with the rook to move, is normally won for the rook except when trivially lost,

but the computer counts five non-trivial losses and 112 draws. The non-trivial losses are given by Re4 against Ba8/Kg6 and Rh1 against Ba8/Kf6, Bd5/Kb7, Bd5/Kf7, or Bd5/Kf3, the rook having no good move in any case and Black being able to sacrifice both his men. However, the position of bK is crucial. White to play loses with Rh1 against Bd5/Kf7, but if bK is on f8 he wins by Rc1 and if bK is on f6 he wins by Ra1.

The draws are all characterized by bK shielding bB, either at the start (Ba8/Kc6 against Rf4) or after the first move. The latter case is typified by Ba8/Kc7 against Re4, where bB is attacking wR. The wR moves away (say 1 Rf4) and Black cannot sacrifice bB without conceding a losing K v R ending; so he shields his bishop, and we have a position of the first kind.

Some of the wins with the rook are also interesting, and one of the longest has Rh3 against Ke5/Bd7. White starts 1 Rh2 (1 Rh1 only draws), and 1...Bh3 concedes a loss with K v R. Black therefore shields his bishop, and 1...Kf5 is his better move. Now White plays 2 Rh1, and Black has a dolorous choice: 2...Bc6 3 Rh7 (only move to win), or 2...Ke6 3 Ra1 (best, though some other moves win more slowly), or 2...Bc8/Be6 3 Ra1 (this time White has alternatives just as good). In each case, White's next move will either sacrifice wR or capture bB with a winning R v K ending.

There is one non-trivial position in which whoever is to move loses (Kf3/Bd5 against Rh1) and one in which the pieces lose with the move but can draw without it (Kf4/Bd6 against Rh1). This latter position was misreported in the original document.

Rook against king and knight offers no non-trivial losses for the rook to move, but there are 1677 draws. To hold the game, the pieces must come close together, and a non-trivial position is always drawn if the king and knight are within two squares of each other and the king is away from the edge. However, there are some opportunist wins for the rook if the king is on the edge, even if the knight is close. A longest such win has Rg3 against Ke8/Nc7, with play

1	Rg3-b3 !	Nc7-e6 !	3	Rb5-g5 !	Nf8-h7 !
2	Rb3-b5 !	Ne6-f8 !	4	Rg5-g4	

and a sacrifice to follow.

If the rook can safely command a line between the pieces, it wins. To get a feel for the material, let us suppose the knight to be on b6, the rook on e1, and the king somewhere in the rectangle g8-h8-h3-g3. If the king is on the eighth or seventh rank, 1 Re5 forces the knight to move to the left; if the king is on the sixth rank, 1 Rc1 forces the knight to the left; if the king is on the fifth or fourth rank, 1 Re7 forces the knight to the left; if the king is on the third rank, 1 Rb1 wins at once. The rook is a more precise attacker than the queen, and there are no draws against widely separated defenders such as occurred with a queen against the same men. The only cases where the rook cannot immediately force the knight to move further away from the king arise when it is on the eighth rank and the king is on the seventh or sixth, and in these positions the rook transfers its attention to the king and gradually cramps it as in the two-man ending R v K. A longest win occurs with Rd5 against Na8/Kf7, with play

1	Rd5-d4	Kf7-f6 !	6	Rf4-f8 !	Nc7-b5 !
2	Rd4-d3	Kf6-g7 !	7	Rf8-e8 !	Kh6-g6 !
3	Rd3-e3 !	Kg7-g6 !	8	Re8-e4	Nb5-a7 !
4	Re3-e4	Kg6-h6	9	Re4-d4 !	Kg6-f6
5	Re4-f4 !	Na8-c7 !	10	Rd4-d8 !	

and a sacrifice next move unless Black sacrifices first. The knight is driven into the open at move 5 (5...Kh7 loses more quickly), and all is then plain sailing.

This ending featured in a 1997 study by Fabrice Liardet which explored some of the winning positions for one White man (rook, bishop, or king) against king and knight.

Rook against two kings is straightforward. Apart from trivial wins for each side, there are some simple wins for the rook if it can crowd both defenders against the edge; otherwise the position is drawn. A longest win occurs with Rd2 against Kb8/Ka5; play 1 Rc2 Ka8 2 Rc3 and either 2...Ka7 3 Rc6 or 2...Ka6 3 Rc7.

There are three non-trivial positions in which the kings lose with the move but can draw without it (Ka8/Kd8 against Rf6, Ka8/Kb7 against Rd5, Kb8/Ka7 against Rd5).

3. Lone bishop

Nearly all endings with **bishop against two line-moving pieces** are trivial, and even the few which are not (for example, Bd1 to move against Qd6/Qd8) can be resolved very quickly. That said, even a short solution may have its charm. A delightful try line in a 1997 study by Fabrice Liardet comes down to wRb6/Rf8 to move against bBe1, and although White can sacrifice either rook he then leaves himself with a lost R v B ending!

Another interesting case is that of bishop against queen and bishop, where there are ten non-trivial positions in which the pieces lose with the move but can draw without it (Qa8/Ba6 against Bd2, Qb8/Bb6 against Bc2 or Be2, Qb8/Bb4 against Bc2, Qc8/Bc6 against Bf2, Qd8/Bd6 against Bc2, Qd8/Bd4 against Bc2, Qa7/Ba5 against Bd1, Qb7/Bb5 against Be1, Qc7/Bc5 against Bf1). In each of these positions, the queen is on a square of the same colour as her bishop, the bishop blocks the queen on the file, and the opposing bishop is on a square of the opposite colour six ranks away.

Bishop against queen and knight offers a single drawing position: Bd1 against Qd6/Nh8. White's only safe move is 1 B a4, Black's only safe reply is 1...Qf4, and we have a reflection of the same position. There are also 228 non-trivial wins for the bishop to move and 25 non-trivial losses, but none is difficult. The longest win has Bb1 against Qb8/Nd8: play 1 Bh7 Qa7 2 Bg6 Nb7 3 Bf7, and Black has no good move.

There is one non-trivial position in which the side to move loses (Qb8/Nb6 against Bb1).

Bishop against rook and knight offers 392 non-trivial wins for the bishop to move and 43 non-trivial losses, but none is difficult. There are no draws.

There are two non-trivial positions in which the side to move loses (Ra8/Na5 against Bb1, Rb8/Nb6 against Bb1).

Bishop against bishop and knight, with unlike bishops, is always trivial; the player with the pieces, if not forced to make an immediate capture, can hold the draw by playing his knight to a square of the opposite colour to that of his bishop and leaving it there. With like bishops, all non-trivial positions are won for the lone bishop.

Bishop against two knights offers 3155 non-trivial wins for the bishop to move and 10 non-trivial losses. A longest win has Bd1 against Nb8/Ne7, with typical play 1 Bh5 Nc8 2 Bf3 Nd7 3 Be4 and the knights are dominated; hardly difficult, but White must avoid 2 Bg4 because 2...Nc6 3 BxN Na7 wins for Black. The non-trivial losses all feature a dominated bishop (for example, Bc1 against Nb7/Nd7).

There are nine non-trivial positions in which the side to move loses. The computer lists them as follows:

Na8/Ne6 v Be1	Nc8/Nf7 v Be1	Nb7/Nd7 v Bc1
	Nc8/Ne6 v Be1	
Nb8/Nd6 v Bd1		Nc7/Ne7 v Bd1
Nb8/Nc5 v Bh5	Nd8/Nd6 v Bd1	
	Nd8/Nf6 v Bf1	

The tenth non-trivial loss with White to move (Bc1 against Nd7/Na2) does not appear in this list because it is trivial with Black to play.

Bishop against king and queen offers 21 non-trivial wins for the bishop to move, 28 non-trivial losses, and 160 non-trivial draws. None is difficult, but in two cases (Bc1 or Be1 against Kd5/Qd7) a losing bishop can delay making a capture until move 4. The draws arise when the bishop can force the queen to sacrifice itself.

There are four positions in which the bishop loses with the move but can draw without it (Kd7/Qa7 against Bh7, Kd6/Qd7 against Bd2, Kd5/Qd7 against Bc1 or Be1) and three in which the same is true of the pieces (Kc8/Qb8 against Bh5, Kc7/Qa7 against Bg6, Kd6/Qd8 against Bc2).

Bishop against king and rook is generally similar, offering 20 non-trivial wins for the bishop to move, 61 non-trivial losses, and 300 non-trivial draws. In two cases, Bc1 or Be1 against Kd5/Rd7, a losing bishop can delay making a capture until move 8. In each case, 1 Bd2 is White's best move, and the computer then plays

1	Rd7-d8	4	Bd2-e1 !	Ke6-d5 !
2	Bd2-c1 !	Kd5-d6 !	5	Be1-d2 !	Rd8-d7 !
3	Bc1-d2 !	Kd6-e6	6	Bd2-e1	Rd7-e7 !

after which any White move allows Black to sacrifice both his men (but not by 7...Re1, which only draws). The play from Black's 1st move to his 5th is another "lose-a-move" manoeuvre.

There is one position (Kd7/Ra7 against Bh7) in which the bishop loses with the move but can draw without it.

Bishop against king and bishop is normally drawn when not trivially won or lost, but there are 35 non-trivial wins for the lone bishop. A longest has Bh7 against Ba8/Kc6, with typical play 1 Bg8 Kb7 2 Bf7 etc.

Bishop against king and knight is complicated in detail (the computer counts 4565 non-trivial wins for the bishop to move, 49 non-trivial losses, and 1107 non-trivial draws) but strategically straightforward.

- If the knight is on a square of the same colour as the bishop, the bishop can normally play to attack squares adjacent to the king, since a sacrifice of the king will leave Black with a losing N v B ending. The king will therefore have to retreat, and the bishop can hope to repeat the process and gradually drive him to the edge of

the board. For example, set Be1 against Nd6/Ke4, when White wins by 1 Bc3 Kf3 2 Bd2 Kg2 3 Be1 Kh1 4 Bf2, or against Ne7/Ke6, when the win is 1 Bc3 (Bg3 also wins) Kd7 2 Be5 Ke8 3 Bd6/Bf6. However, if the knight is on one of the central 16 squares and the king is next to it, the king can run round the knight indefinitely and Black draws (for example, Be1 against Nd6/Kd5, play 1 Bc3 Kc6 2 Bd4 Kd7 3 Bb6 Ke6 and so on). White must also be careful lest the king sacrifice himself on c8 or d8 and come down to one of the exceptional *winning* N v B endings. Set Be1 against Nd6/Kf5 (our first example with bKf5 instead of e4), and 1 Bc3 fails; Black replies 1...Kg4 2 Bd2 Kh3, and now 3 Be1 actually loses (3...Kh4 4 Bxh4 Nc4). The only win is 1 Bh4 Ke4 2 Bf6 Kf3 3 Bg5 Kg2 4 Bh4 Kh1 5 Bf2, driving bK to h1 as before.

- If the knight is on a square of the opposite colour, advancing on the king is potentially dangerous, because Black may be able to sacrifice king and knight on successive moves. The bishop may even be dominated, any move leaving him liable to capture the knight. White's winning chances in this case are therefore few, and he may be hard pressed even to draw.

The computer gives Bd2 against Ke7/Na8 as a longest win for the bishop. Here the knight is on a square of the opposite colour to the bishop, but it is poorly placed in the corner and White can win in spite of this disadvantage. An optimal line of play is

1	Bd2-c3 !	Ke7-d7 !	4	Bb2-f6 !	Kb5-a6
2	Bc3-a1	Kd7-c6	5	Bf6-e7 !	Ka6-a7
3	Ba1-b2	Kc6-b5	6	Be7-c5 !	

with a striking final position (6...Nc7 7 BxK will be a win for the bishop). Several other waiting moves are as good at moves 2 and 3.

A longest win for the king and knight has Kd8/Nd7 against Bc1, with play

1	Bc1-d2 !	Kd8-c7 !	3	Be1-d2 !	Kd6-d5 !
2	Bd2-e1	Kc7-d6 !	4	Bd2-e1	Kd5-e4 !

and two sacrifices to follow. Here, even a square as distant from the centre as d7 allows the knight to exert a decisively cramping influence on the bishop, keeping it at bay until the king can come and help. White can play Bc1 instead of Be1 at moves 2 and 4, but the outcome is the same.

There are ten non-trivial positions in which the bishop loses with the move but can draw without it (Kd8/Nd7 against Bc1 or Be1, Kc7/Nd7 against Bd2, Kd6/Nd7 against Be1, Kd6/Nf7 against Be1, Kd5/Ne8 against Bd2, Kd5/Nf8 against Bd1, Kd5/Nd7 against Bd2, Kd5/Nh5 against Ba5, Kd5/Ng4 against Ba5) and five in which the same is true of the pieces (Ka8/Nf5 against Ba5, Ka8/Ng4 against Ba5, Ka8/Nh3 against Ba5, Kc8/Ne5 against Bc5, Kd8/Nf5 against Bd5).

This ending was examined by Fabrice Liardet in 1991 and 1997.

Bishop against two kings is normally drawn when not trivially won or lost, but there are 179 non-trivial wins for the bishop (mostly by threatening an immediate sacrifice which Black cannot evade) and 67 for the kings. No win for the bishop takes more than three moves, the longest having Ka8/Kb8 against a bishop able to play to a4 or e8; typical play is 1 Ba4 and either 1...Kaa7 2 Bc6 or 1...Kba7 2 Bd7.

In the absence of a quick win for the bishop, it is the kings which have such winning chances as exist, but the bishop can hold the draw if it can safely move to the long or an adjacent diagonal; it simply patrols this diagonal, and neither king can approach without allowing the bishop to sacrifice itself. The non-trivial wins for the kings all start with the bishop on c1, d2, e1, or an equivalent square. All such positions are won for the kings unless the bishop can sacrifice itself or move immediately to the long or an adjacent diagonal, the kings always being able to advance to a position of domination. A longest win has Kd8/Kd6 against Bd2, with typical play 1 Bh6 K8d7 2 Bc1 K7e6 3 Bd2 Ked5 4 Bc1 Kc4 5 Bh6 Kb3 (quicker than the apparently natural 5...Ke6) and the bishop must allow two sacrifices.

There is one non-trivial position (Kd8/Ka5 against Bd5) in which the kings lose with the move but can draw without it.

4. Lone knight

The knight is the weakest of the pieces in one-against-one endings (knight against knight depends on the move, knight against any other piece is normally a loss) and endings with a lone knight against two pieces are usually good for the pieces; unless the lone knight has a trivial win, or can threaten a sacrifice which the opponent cannot evade, the player with the two pieces cannot normally be prevented from sacrificing one of them and coming down to a winning ending with the other. In particular, all positions with **knight against two line-moving pieces** come into this category. There are however 13 non-trivial positions with knight against rook and bishop in which whoever is to move loses (Ba8/Rb7 against Nf4, Nh4, Ng3, or Nh2, Bb8/Rc7 against Ng4, Nf3,

Nh3, Ng2, Nf1, or Nh1, and Bc8/Rd7 against Ng3, Nh2, or Ng1) and one such position with knight against two like bishops (Bc8/Bh3 against Nc3, an interesting variation on the win by domination with knight against one bishop). The positions with knight against rook and bishop all feature a bishop masked by its rook.

Knight against queen and knight is broadly similar, but the second player can sacrifice the queen only when the resulting N v N ending will be won and this adds a complication. There are 1109 non-trivial wins for the lone knight to move, though only 24 of them take more than two moves. One of the longest (four moves) has Nh2 against Qb8/Nd6; play 1 Ng4 (only move to win) Qb4 (best) 2 Nf2 (threatening to sacrifice on e4, since if 2...Qg4 then 3 NxQ gives White a winning N v N ending) Nc4 (only move to prevent the sacrifice on e4) 3 Ne4, with a winning sacrifice or capture next move.

There are three non-trivial positions in which the side to move loses (Qa8/Nc6 against Nf4, Qb8/Nd6 against Ng4, Qc8/Nc5 against Ne2).

Knight against rook and knight is similar but somewhat easier; there are 1197 non-trivial wins for the knight to move, but only six of them take more than two moves (Ng7, Ng5, Nf4, or Ne3 against Ra8/Nb8, Nh3 or Ng2 against Rc7/Na8). In the first group, we have 1 Ne6/Nd5 Ra1 (say) 2 Nc7 threatening to sacrifice both on a8 and on a6, and 2...Ra8 3 NxR gives the N v N win to White. In the second group, we have 1 Nf4 Rc1 (say) 2 Ne6 threatening to sacrifice on c7, and again a rook sacrifice gives the N v N win to White.

There are three non-trivial positions in which the side to move loses (Ra8/Nb8 against Ne6 or Nd5, Rc7/Na8 against Nf4).

Knight against bishop and knight is statistically complex (the computer counts 4483 non-trivial wins for the knight to move, 188 of them taking more than two moves, 7281 non-trivial losses, and 507 draws), and although there is a strategic plan for Black which normally works there are many positions in which some detailed analysis is necessary.

Black's plan is to place his knight on a square of the same colour as that of his bishop and then to play as if with B v N alone. If the White knight ventures within range of the Black, Black sacrifices his knight and wins with B v N; if White avoids this, Black rounds off the B v N fight by sacrificing his bishop, and this gives him a winning N v N ending.

The most important cases in which this procedure does not work feature Bc8 against a White knight able to play 1 Na7 and Bd8 against a knight able to play 1 Nd3. The former always gives White a win. In the latter, Black wins quite quickly if his knight is on d6 (he can play ...Bc7/Be7 getting his bishop away from the edge), but otherwise his bishop is dominated and his only hope of survival is to play his knight to a square four moves away from d3; in fact ...Nf5 gives him a win and ...Na8/Ne8/Nb5/Nb1/Nf1 hold the draw, but ...Nc8/Ng8/Nh7 lose. Suppose Bd8/Nf8 to play against Nd3. The only move to delay defeat is 1...Nh7; White plays 2 Nf4, and another knight move will lose immediately; of the bishop moves, 2...Bc7/Bb6/Ba5/Bg5 are met by 3 Ng2 with no escape square for the bishop (if it sacrifices itself, White will win the N v N ending), 2...Be7 by 3 Ne6, and 2...Bf6/Bh4 by 3 Nh3.

There are in fact 55 draws with the Black knight and bishop on squares of the same colour, listed by the computer as follows:

Ba8/Nb7 v Nf8, h8, e7, g7, f6, h6, e5, f4, h4, g3, h2	Bd8/Nb6 v Nf2, c1, e1
Ba8/Nc6 v Nf6, g5, g3	Bd8/Nf6 v Nb2, c1, e1
Bc8/Ne6 v Na2, b1, d1	Bd8/Na3 v Nc5, f4, f2, c1
Bc8/Nf5 v Na4, a2, b1	Bd8/Nc3 v Ne5, e1
Bd8/Na7 v Nc5, f4, b2, f2, c1, e1	Bd8/Nd2 v Nb4, f4
Bd8/Nc7 v Ne5, b2, f2, a1, c1, e1	Bd8/Nh2 v Nc5, b4, f4, b2, c1
	Bb7/Nc6 v Nh5, e4, h3, h1

Most of these draws depend on an opening move for White which prevents the bishop from moving and so forces Black to play his knight to a square of the wrong colour (1 Nd3 against Bd8/N--, 1 Nc2 against Bd8/Nc7, 1 Nc3 against Bc8/Ne6 or Bc8/Nf5). The others feature a Black knight on c6 shielding a bishop oscillating between a8 and b7. Here (suppose Nh1 against Bb7/Nc6 to fix our ideas) we have 1 Ng3 Ba8 2 Ne4 and the knight must move since 2...Bb7 allows 3 Ng3 repeating the position, and although it can get back on the right colour (2...Nd8 3 Nf6 Nc6) Black is making no progress.

All other non-trivial positions with bishop and knight on squares of the same colour are wins for Black. The computer gives Bb8/Nc7 against Na1 as one of the positions where the win takes longest, with play

- Three-man pawnless endings in Losing Chess (July 1998) -

1	Na1-c2 !	Nc7-e8 !	6	Nd1-e3 !	Bd6-b8 !
2	Nc2-e3 !	Bb8-d6 !	7	Ne3-c4	Ne8-c7 !
3	Ne3-g2	Bd6-f8 !	8	Nc4-e5	Bb8-a7 !
4	Ng2-e3 !	Bf8-a3 !	9	Ne5-f7	Ba7-f2
5	Ne3-d1 !	Ba3-d6 !	10	Nf7-h8	Bf2-d4

and a sacrifice next move. If it were White to move after 1 Nc2 Black would win quickly, but as it is his knight has to play to a light square and this delays matters. White's subsequent threats to sacrifice to the bishop, alternating with moves to the dominant square e3, prevent the knight's immediate return to a dark square, but Black can eventually break the cycle by the shielding manoeuvre ...Bb8 and...Nc7. The position after 7...Nc7 isn't quite the same as that after 1 Nc2, since the White knight is on c4 instead of c2, but it makes no difference.

If the Black knight is on a square of the wrong colour, everything depends on whether it can play to a square of the right colour, and detailed analysis may well be necessary. The computer gives Ng1 against Bc1/Na8 as one of the longest wins for the lone knight, with play

1	Ng1-f3 !	Bc1-a3 !	4	Nb3-c5 !	Bg3-b8
2	Nf3-d2 !	Ba3-d6 !	5	Nc5-e6 !	
3	Nd2-b3 !	Bd6-g3			

and a sacrifice next move. Black cannot play 2...Nc7/Nb6 on account of 3 Nb1, since 3...N-- 4 NxB would win for White, so he hopes for the shielding manoeuvre ...Bb8 and ...Nc7 as before; but if he plays 3...Bb8 then White can reply 4 Nc5, so he must waste a move, and this allows the White knight to get too close.

There are 40 positions in which the side to move loses, listed by the computer as follows:

Ba8/Nb7 v Nb8, g5	Bc8/Nd7 v Nd4, f4, e3, g3, f2, h2,	Bd8/Nh8 v Nd3
Bb8/Na8 v Na5, d2	g1	Bd8/Ne7 v Ne4, f3, g2
Bb8/Ne8 v Nc5	Bc8/Nh3 v Nc3	Bd8/Na5 v Nd3
Bb8/Nc7 v Nb7, f5, a4, c4, e4, g4, d3, f3, h3, e2, g2, f1, h1	Bc8/Ne1 v Nc4	Bd8/Ng5 v Nd3
Bb8/Nd1 v Nb4	Bd8/Nb8 v Nd3	Bd8/Na1 v Nd3
	Bd8/Nf8 v Nd3	Bd8/Ng1 v Nd3
	Bd8/Ng8 v Nf2	Bc7/Na6 v Nf6

There are also 33 positions in which the knight loses with the move but can draw without it,

Ba8/Nc6 v Ne4	Bd8/Na8 v Nb2, c1	Bd8/Nd2 v Nd3
Bb8/Na8 v Ne3, b2	Bd8/Nc8 v Nf2	Bd8/Nh2 v Nd3
Bb8/Ne8 v Ng5	Bd8/Nh7 v Ne1	Bd8/Nb1 v Nb4, b2, c1
Bb8/Na4 v Nb4	Bd8/Na7 v Nd3	Bd8/Nf1 v Nf4, f2
Bc8/Nb8 v Nc2	Bd8/Nc7 v Nd3, c2	Bb7/Nf8 v Nf7
Bc8/Ne6 v Nc3	Bd8/Na6 v Nc1	Bb7/Nc6 v Ng3
Bc8/Nf5 v Nc3	Bd8/Nb6 v Nd3	Bd7/Na3 v Nb3
Bc8/Nh2 v Nc2	Bd8/Nf6 v Nd3	Bd8/Na4 v Nb4
	Bd8/Na3 v Nd3	Bd8/Na3 v Nd3
	Bd8/Nc3 v Nd3	Bd6/Na2 v Nb2

and 10 positions in which the same is true of the pieces:

Ba8/Nd8 v Nd7, b5	Ba8/Nb7 v Ne7, f6, e5, f4, h4, g3, h2	Bd8/Nf1 v Nd4
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This ending was examined by Fabrice Liardet in 1991 and 1997.

Knight against two knights is one of the classic Losing Chess endings. The simplicity of knight against knight, where the result depends only on a like/unlike relation between square colours, inevitably led analysts to seek a similar resolution of multiple-knight endings, but in fact no such resolution appears to exist; even the simplest such ending, one knight against two, does not lend itself to easy characterization. There are 3290 non-trivial wins for the single knight to move, 372 non-trivial losses, and 3062 draws. K. Fabel examined the ending in 1947 and F. Hansson in 1948, and it was definitively analysed by Gyorgy Evseev in 1992.

The non-trivial losses can be disposed of quickly. Each depends on domination of the single knight; the two knights must be on unlike squares, and the single knight must be unable to move without allowing one of the two knights to sacrifice itself. If the single knight can make even a single safe move, it can avoid defeat.

To get a feel for the single-knight wins, let us suppose that Black has just moved, and that his knights are now on

squares of opposite colour (this will always happen after alternative Black moves). If one of his knights is now one move away from White's, White will take it and win the resulting N v N ending; if it is two moves away, White will sacrifice to it. His knights must therefore be at least 3 and 4 moves away from White's. To fix our ideas, let us put the White knight on b5 and the Black knights on b8 and d7.

- If White now approaches the nearer knight (b8), putting himself only two moves away from it, Black will sacrifice and win the N v N ending. (We measure distances in knight moves, so b8 is nearer than d7.) White must therefore retreat from this knight, and put himself at least 4 moves away from it. Since White is trying to win, let us assume that he approaches the other knight, putting himself only 3 moves away from it: 1 Nd6.
- If Black now advances his more distant knight (now b8) and puts it only 3 moves from the White knight, White may be able to play so as to be only 2 moves from each Black knight, and this gives him a win by "fork" (here, for example, 1...Nc6 can be met by 2 Nc8, giving Nc8 against Nc6/Nd7). In such a position, White threatens to sacrifice to each Black knight, and if either sacrifices first White wins the N v N ending. Black should therefore leave his more distant knight alone and move his other knight, retreating so as to leave it also 4 moves away from White's: 1...Nf8.
- White's natural reply is to attack, putting his knight 3 moves away from each of Black's: 2 Nf7, say.
- Black must now retreat, since to put either knight only 2 moves away from White's will lose immediately. If he cannot, White has a win by domination. But if either Black knight can retreat, as here by 2...Na6, we have another position where one knight is 3 moves away from White's and the other is 4 moves away, and this is where we came in. If Black can keep this up indefinitely, he will draw.

Examination of computer output shows that Black can hold the draw if at any time he can place his knights on squares of unlike colour within the central 16 squares without allowing an immediate sacrifice or a capture, or on squares of like colour within the region c7-f7-g6-g3-f2-c2-b3-b6 without allowing an immediate sacrifice or a fork. There are also some other draws, the example above (which is in fact drawn) being a case in point. The longest win has Nb1 against Nh8/Na6, when the computer gives the following:

1	Nb1-c3 !	Na6-b8 !	6	Nc5-e6 !	Na3-c4 !
2	Nc3-d1 !	Nb8-a6 !	7	Ne6-d4 !	Nc4-b2
3	Nd1-f2	Na6-c7 !	8	Nd4-c6	Nb2-d1
4	Nf2-d3 !	Nc7-b5 !	9	Nc6-b4 !	Nh8-f7
5	Nd3-c5 !	Nb5-a3 !	10	Nb4-d3 !	

with a win by fork. There are alternatives for Black from move 7 onwards, but all lead to much the same finish. The purpose of the surprising move 2 Nd1 is to get the White knight to d3 (at move 3, Nb3 is as good as Nf2) without allowing Black to extricate his knight from h8. If White plays the natural 2 Nd5, Black can hold the draw by 2...Nf7; if after 2 Nd1 he tries 2...Nf7, White can play 3 Nf2 with a fork or domination next move.

As might be expected in an ending featuring knights alone, there are a large number of positions of reciprocal zugzwang. There are only 10 non-trivial positions in which the side to move loses,

Na8/Ne7 v Nh5	Na8/Nh4 v Nf1	Nb8/Nc6 v Na8
Na8/Ne5 v Ng8, b7, g2	Na8/Ng3 v Ng8	Nb8/Nd1 v Na3
Na8/Nf4 v Na2		Nb7/Nd6 v Nb8

but there are 362 positions in which the single knight loses with the move but can draw without it, and 239 positions in which the same is true of the two knights.

Knight against king and queen brings us back to positions which if not trivial wins for the knight are normally wins for Black. Here the computer counts 132 non-trivial wins for the knight, but most involve a threatened sacrifice which Black cannot evade and the rest depend on a mechanism exemplified by Nb1 against Kc5/Qc8: 1 Nd2 (threatening a sacrifice on c4) Kc6 (the only way to prevent this sacrifice) 2 Nb3 (making the same threat on c5) Kc7 3 Na5 (making the same threat on c6, and this time there is no defence). Note that the blindly systematic move 3 Nd4 does not work on account of 3...Qe6. This mechanism was exploited in a 1980 study by Jörg Kuhlmann (with a rook instead of a queen), and recently by Paul Byway.

Knight against king and rook is similar. The computer counts 428 non-trivial wins for the knight, of the same character as those with knight against king and queen. Again the mechanism has been exploited by Jörg Kuhlmann and Paul Byway.

Knight against king and bishop is even simpler: there are 269 non-trivial wins for the knight, but all involve an immediate threat to sacrifice which Black cannot evade. There are eight positions in which the side to move loses (Ba8/Kb7 against Nf6 or Nf4, Bb8/Kc7 against Ng4, Nd3, or Nf3, Bc8/Kd7 against Ne3 or Ng3, and Bd8/Ke7 against Nf3). All feature a bishop masked by its king.

Black's possession of an extra king in **knight against king and knight** gives him the invaluable ability to make a tempo move, and his strategy is to play a waiting move with his king if necessary, to win the N v N fight without allowing wN to sacrifice itself to bK, and then to win with K v N. However, the computer counts 1401 non-trivial wins for the knight, and 57 draws. The wins for the knight all feature a Black king which is open to harassment, and the longest has Nc2 against Ka8/Nh3: 1 Na3 (only move to win) Ka7 (best, if say 1...Nf4 2 Nc4 N-- then 3 Nb6 and 3...N-- 4 NxK will win for White) 2 Nc4 (again the only move to win) Kb8 (best) 3 Ne5 (yet again the only move to win) Ka7 (3...Ka8 is as good, but if 3...N-- then 4 Nd7 etc) 4 Nd7 Kb8/Kb6 5 NxK.

The draws are similar in character, but White, although able by harassing the king to prevent Black from reaching the normal win, is unable to clinch the victory himself. The computer lists them as follows:

Ka8/Nh7 v Ne3, b2, c2, d2, b1	Kc8/Na8 v Nh8, h5, h4, d3, h3, g2
Ka8/Nh5 v Ne3, b2, c2, d2, b1	Kc8/Nc1 v Ng8, h7, h5, e4, g4
Kb8/Nh7 v Nc4, d3	Kd8/Na8 v Nh8, h7, e4, f4, f3
Kb8/Nh5 v Nf7, c4, f3	Kd8/Nh8 v Na3, e3, b2, d2
Kb8/Ng3 v Na3, b2	Kb7/Nh6 v Nc3, d2
Kb8/Nb1 v Nf8, g7, g5, d4, f4	Kc7/Na8 v Ng6, g5, h5, d3, g3, h3, f2, g2
Kb8/Nd1 v Nf8, g7, g5, d4, f4	

There are 82 non-trivial positions in which the side to move loses:

Ka8/Nb8 v Ne6, d5, a4	Kb8/Na8 v Ne5, b4	Kc8/Nb8 v Nf5
Ka8/Nd8 v Nd5, a4, c4	Kb8/Nc8 v Nf8, f6, e5	Kc8/Na7 v Nc4
Ka8/Nf8 v Nd5, a4	Kb8/Nb5 v Nf8, f6, e5	
Ka8/Nh8 v Ne8, d5	Kb8/Nf5 v Nf8, f6, e5	Kd8/Nc8 v Ng5
Ka8/Nb7 v Ne7, f6, f4	Kb8/Na4 v Nb4	Kd8/Ne8 v Na5
Ka8/Nf7 v Ne7, f6, f4, c3, e3	Kb8/Nh3 v Nf8, f6, e5	Kb7/Nf8 v Nf7
Ka8/Ng7 v Nd5, a4	Kb8/Na2 v Nf8, f6, e5	Kb7/Nh2 v Ne4
Ka8/Nd6 v Nd5	Kb8/Nc2 v Nf8, f6, e5	
Ka8/Nh6 v Ne8, d5	Kb8/Ne2 v Nf8, f6, e5	Kc7/Nh3 v Nc3
Ka8/Ne5 v Ne8, e6, d5	Kb8/Ng2 v Nf8, f6, e5	Kc7/Nh1 v Nc3
Ka8/Ng5 v Nd5, a4	Kb8/Nb1 v Nf6, e5	
Ka8/Nh4 v Ne8, d5	Kb8/Nd1 v Nf6, e5, b4	Kd7/Nh8 v Nd3
Ka8/Nf3 v Nf6, f4	Kb8/Nf1 v Nf6, e5	Kd7/Na1 v Nd3
Ka8/Ng3 v Ne8, d5, a4	Kb8/Nh1 v Ne5	Kd7/Ng1 v Nd3
Ka8/Nh2 v Nd5		

The last group is perhaps the most interesting. Suppose Kd7/Nh8 against Nd3. White to move will probably try 1 Nb4 hoping for 1...Kc6 2 NxK with a win, but if Black retreats by 1...Ke8 White is helpless. Black to move cannot safely move his knight, and if he tries say 1...Kc7 White replies 2 Nc5, threatening to sacrifice on a White square and still winning if Black sacrifices first. Black's best is therefore 1...Ke7 or 1...Ke8, ready to meet 2 Nc5 by 2...Kf8, but White can follow up by 3 Nd7 and 3...N-- 4 NxK will again give the win to White.

There are also four non-trivial positions in which the knight loses with the move but can draw without it (Kb8/Nh7 against Ne5, Kb8/Nh5 against Ne5, Kd8/Na8 against Ng5, Kb7/Nh6 against Ne4) and three in which the same is true of the pieces (Kb8/Nb1 against Nf8, Kb8/Nd1 against Nf8, Kd8/Na8 against Nf4).

Knight against two kings is yet again similar, Black's strategy being to sacrifice one king and then the other, but the computer counts 214 non-trivial wins for the knight to move. A longest has Nh8 against Kc8/Kd7, with play 1 Ng6 Kc6/Kdc7 (1...Ke7 2 NxK is a win for White, so Kd7 must retreat) 2 Nf8 and sacrifices on d7 next move. A 1997 study by Fabrice Liardet (currently awaiting publication) has the same finish.

5. Lone king

King against two queens and king against two rooks are easily summarized: if the king, having the move, cannot sacrifice itself on the first or second move, it loses. In the case of king against two queens, there is one non-trivial position in which the side to move loses (Qb8/Qf8 against Kd3).

King against queen and rook is similar, but the computer identifies eight draws: Kf4 or Kf3 against Qa8/Rc6, and K in the triangle e4-g4-g2 against Qb7/Rc6. Suppose the first. White plays 1 Ke4, Black's only safe reply is 1...Qb7, and we have a position in the second group. White now plays 2 Kf4 or Kf3, Black must go back again, 2...Qa8, and so on. But White must be precise. If he plays 1 Kf3 against Qa8/Rc6, Black can reply 1...Qa1 and disentangle himself; if he plays 1 Kf4, Black can reply 1...Qa2. There is one non-trivial position in which the side to move loses (Qc8/Rd7 against Kf5), and one in which the king loses with the move but can

draw without it (Qa8/Rc6 against Ke4).

King against queen and bishop features 175 non-trivial wins for the king to move and 292 non-trivial draws, but nearly all the wins have the queen and bishop on the same diagonal in such a position that a threat to sacrifice on it cannot be countered. There are also a few non-trivial wins against Qc8/Bc7 or Qd8/Bd7. The longest have Kc4, Ka3, Ka2, Kb2, or Kc2 against Qc8/Bc7, play proceeding 1 Kb3 Qd8 (only move to avoid immediate loss) 2 Ka4/Kb4 with a sacrifice next move. If the king starts on c3, 1 Kc4 wins more quickly.

If the king cannot win, the pieces will try to do so by sacrificing the bishop before the queen (or sacrificing the queen in such a way that the bishop will immediately follow). However, success is not certain, and the computer lists the following positions as drawn (the king to play):

Ba8/Qb8 v Kg6	Bc8/Qh7 v Kc5, a4, b4, a3, e1
Ba8/Qc8 v Kh6	Bc8/Qa6 v Kc1
Ba8/Qc7 v Kh5	Bc8/Qb6 v Kd1
Ba8/Qg3 v Ke8	Bc8/Qc6 v Ka2, a1
Ba8/Qh3 v Kf8	Bc8/Qd6 v Ka1, b1
Ba8/Qh2 v Kf7	Bc8/Qe6 v Kc1
	Bc8/Qf6 v Ka4, d1
Bb8/Qc8 v Kh6, a4, a3, a2, e2	Bc8/Qg6 v Ka4, b4, a3, e1
Bb8/Qd8 v Ka3, b3	Bc8/Qh6 v Kb4, c4, a3, b3, a2, a1, f1
Bb8/Qe8 v Kc3, a2, b2, c2	Bc8/Qa5 v Kg1
Bb8/Qf8 v Kd3, c2, d2	Bc8/Qc5 v Ka1
Bb8/Qg8 v Kd2, e2, c1, d1	Bc8/Qf5 v Ka3
Bb8/Qh8 v Ke2, d1, e1	Bc8/Qg5 v Ka3, b3, a2, a1
Bb8/Qa7 v Kc2, c1	Bc8/Qh5 v Kb3, c3, b2, b1
Bb8/Qc7 v Ka3, a2	Bc8/Qa4 v Kf2, f1
Bb8/Qd7 v Ka2, b2	Bc8/Qb4 v Kg1
Bb8/Qe7 v Kc2	Bc8/Qf4 v Ka2, a1
Bb8/Qf7 v Kd2, c1, d1	Bc8/Qg4 v Ka2, b2, b1
Bb8/Qg7 v Ka4, d1	Bc8/Qh4 v Kb2, c2, c1
Bb8/Qa6 v Kc1	Bc8/Qe3 v Ka1
Bb8/Qb6 v Kd1	Bc8/Qf3 v Ka1, b1
Bb8/Qc6 v Ka2, a1	Bc8/Qg3 v Ka1, b1, c1
Bb8/Qd6 v Kb1	Bc8/Qh3 v Ka1, b1, c1, d1
Bb8/Qe6 v Kc1	Bc8/Qf2 v Ka4
Bb8/Qf6 v Ka4, d1	Bc8/Qg2 v Ka4, b4
Bb8/Qg6 v Ka4, b4, a3, e1	Bc8/Qh2 v Ka4, b4, c4
Bb8/Qh6 v Kb3	Bc8/Qf1 v Ka3
Bb8/Qc5 v Ka1	Bc8/Qg1 v Ka3, b3
Bb8/Qf5 v Ka3	Bc8/Qh1 v Ka3, b3, c3
Bb8/Qg5 v Kb3	
Bb8/Qh5 v Kb3, c3, b2, b1	Bd8/Qa8 v Kc3, c2, d2, c1, d1, e1
Bb8/Qa4 v Kf1	Bd8/Qb8 v Kd3, e2, d1, e1
Bb8/Qf4 v Ka2	Bd8/Qc8 v Ka3, e3, a2, e2
Bb8/Qg4 v Kb2, b1	Bd8/Qe8 v Kg2
Bb8/Qh4 v Kc2, c1	Bd8/Qg8 v Ke3
Bb8/Qe3 v Ka1	Bd8/Qh8 v Kf3, e2, f2
Bb8/Qf3 v Ka1, b1	Bd8/Qa7 v Kc2, d1
Bb8/Qg3 v Kb1, c1	Bd8/Qb7 v Kd2, d1, e1
Bb8/Qh3 v Kf8, b1, c1, d1	Bd8/Qc7 v Ka2, e2
Bb8/Qf2 v Ka4	Bd8/Qd7 v Kf2, f1
Bb8/Qf1 v Ka3	Bd8/Qe7 v Kg1, h1
Bb8/Qg1 v Ka3, b3	Bd8/Qg7 v Ke2
	Bd8/Qh7 v Kf2
Bc8/Qa8 v Kc3, c2	Bd8/Qa6 v Kc1
Bc8/Qd8 v Kb3, b2, f2	Bd8/Qb6 v Kd1
Bc8/Qe8 v Kc3, a2, b2, c2	Bd8/Qc6 v Ke1
Bc8/Qf8 v Kd3, c2, d2, a1, b1, c1	Bd8/Qd6 v Kf1
Bc8/Qg8 v Ke3, d2, e2, c1, d1, e1	Bd8/Qf6 v Kd1, h1
Bc8/Qh8 v Ka4, e2, f2, d1, e1	Bd8/Qh6 v Kc4, b3, f1
Bc8/Qa7 v Kc2	Bd8/Qa5 v Kf3, g2, g1
Bc8/Qb7 v Kd2, d1	Bd8/Qg5 v Kb3
Bc8/Qc7 v Ka3, a2, a1	Bd8/Qh5 v Kc3
Bc8/Qd7 v Ka2, b2, b1	Bd8/Qa4 v Kf2, f1
Bc8/Qe7 v Kc2, a1, b1, c1	Bd8/Qb4 v Kg2, g1
Bc8/Qf7 v Kd2, c1, d1	Bd8/Qg4 v Kb2, b1
Bc8/Qg7 v Ka4, e2, d1, e1	Bd8/Qh4 v Kc2, c1

- Three-man pawnless endings in Losing Chess (July 1998) -

Bd8/Qa3 v Ke1, f1	Bc7/Qh5 v Kc3
Bd8/Qg3 v Kb1, c1	Bc7/Qa4 v Kf1
Bd8/Qh3 v Kb1, c1, d1	Bc7/Qg4 v Kb2, b1
Bd8/Qh2 v Kc4	Bc7/Qh4 v Kc2, c1
Bd8/Qa1 v Kf3	Bc7/Qg3 v Kb1, c1
Bd8/Qf1 v Ka3	Bc7/Qh3 v Kf8, b1, c1, d1
Bd8/Qg1 v Ka3, b3	Bc7/Qf1 v Ka3
Bd8/Qh1 v Kb3, c3	Bc7/Qg1 v Ka3, b3
Bb7/Qc8 v Kh6	
Bb7/Qh3 v Kf8	Bd7/Qg8 v Kd2, e2
	Bd7/Qc6 v Ka1
Bc7/Qa7 v Kc2, c1	Bd7/Qh5 v Kc3
Bc7/Qa6 v Kc1	Bd7/Qa4 v Kf2, f1
Bc7/Qb6 v Kd1	Bd7/Qb4 v Kg1
Bc7/Qg5 v Kb3	Bd7/Qh3 v Kc1, d1

These have been grouped by the bishop's square in order to demonstrate their dependence thereon: there are relatively few draws with the bishop away from the edge, and none at all with the bishop on one of the central 16 squares. There are also no draws with the bishop away from the edge and the queen on a corner square, nor with the bishop away from the edge, the king in the angle below it, and the queen hidden above it.

All this suggests a winning procedure for Black: he should get his bishop away from the edge, hide the queen behind it, and gradually advance the bishop towards the White king. A longest win for Black has Qa7/Bd8 against Kc1, with play

1 Kc1-b2 !	Qa7-d7 !	9 Kb2-c2	Bg3-d6 !
2 Kb2-c3	Qd7-e8 !	10 Kc2-d2 !	Qh4-d8 !
3 Kc3-d3 !	Qe8-f8 !	11 Kd2-d3 !	Qd8-d7 !
4 Kd3-d2 !	Bd8-c7 !	12 Kd3-e2	Qd7-c8 !
5 Kd2-d3 !	Bc7-g3 !	13 Ke2-d1 !	Bd6-c5
6 Kd3-c3 !	Qf8-a8 !	14 Kd1-c2 !	Qc8-a8 !
7 Kc3-b3 !	Qa8-h1 !	15 Kc2-c1	Qa8-a6 !
8 Kb3-b2 !	Qh1-h4 !		

and a pair of sacrifices to follow. The Black queen spends the first few moves evading the White king, and then the procedure becomes apparent; the bishop gets away from the edge at move 4, the queen uses the corners as temporary refuges at moves 6 and 7, and Black hides Qd8 above Bd6 at move 10 and Qc8 above Bc5 at move 13. However, the winning procedure is less than straightforward, and a fair amount of detailed analysis may be needed to ensure that progress is being made.

There is one non-trivial position in which the side to move loses (Bd7/Qd8 against Kd4), and two in which the king loses with the move but can draw without it (Bd8/Qb8 against Kd2, Bd7/Qg8 against Ke3).

This ending was exploited in a 1980 study by Jörg Kuhlmann.

King against rook and bishop is easier. There are only 45 non-trivial wins for the king to move, and only 37 non-trivial draws. The draws fall into three clearly defined classes.

- The rook is cramped by the bishop, and the king can force it to sacrifice itself. Typically, we have Kc2 against Ra8/Bd8, with play 1 Kb2 etc (but not 1 Kb3, when 1...Ra4 sacrifices both men, nor 1 Kb1, when 1...Ba5 shields the rook without allowing the king to sacrifice itself to the bishop).
- Kc6 against Ra8/Bg6 (say), when 1 Kb7 forces the rook to sacrifice itself. The same thing works against a bishop on h7 or h5, but not against a dark-square bishop because Black can wait for the capture and then sacrifice on a7 or b8. Neither does it work against a rook on b8 (say Kd6 against Rb8/Bh6), because Black can wait for the capture and then play ...Ba7.
- Rc6, Ba8 or Bb7, and wK within the triangle e4-h4-h1 against Ba8 and the region e4-g4-h3-h1 against Bb7. Kh4 doesn't draw against Rc6/Bb7 because the king must play to the long diagonal (otherwise the bishop will be able to escape to one side or the other).

With these exceptions, Black can win by sacrificing the bishop before the rook, as was pointed out by Klüver in 1934.

King against two bishops is straightforward although often lengthy. With unlike bishops, there are no non-trivial wins for the king, and although the computer counts a large number of draws they all require the king to sacrifice itself while the bishops are separated. If the bishops can achieve a position where they are standing side by side, they win. The computer gives Bd8/Bg6 against Ka3 as a longest win, with play

- Three-man pawnless endings in Losing Chess (July 1998) -

1	Ka3-b3 !	Bg6-h5 !	12	Kd3-e2 !	Bd7-e6 !
2	Kb3-c2 !	Bh5-e8 !	13	Ke2-e1	Bd6-e5
3	Kc2-d3	Be8-d7	14	Ke1-e2 !	Be5-f6 !
4	Kd3-d4 !	Bd7-c8	15	Ke2-d1	Bf6-g7
5	Kd4-e3 !	Bc8-h3 !	16	Kd1-e1 !	Be6-f5 !
6	Ke3-d4	Bh3-d7 !	17	Ke1-f2 !	Bg7-f6 !
7	Kd4-e3 !	Bd8-e7 !	18	Kf2-g1 !	Bf6-g5 !
8	Ke3-d2	Be7-d6	19	Kg1-h1 !	Bf5-g4
9	Kd2-d3 !	Bd6-c7 !	20	Kh1-g1 !	Bg5-d2 !
10	Kd3-e2	Bc7-b8	21	Kg1-h1 !	Bd2-e1 !
11	Ke2-d3 !	Bb8-d6 !			

followed by two sacrifices. The bishops achieve their side-by-side position at move 2 and then advance steadily on the king, but the play at moves 8 and 9 should be noted. After Black's 7th move, the bishops are standing at d7 and e7 and White's natural 8th move is to crowd them by playing Kd3, but this allows the reply ...Bd6 forcing the king to retreat. White's actual move allows him to meet ...Bd6 by Kd3 (or ...Be6 by Ke3) and now it is Black's turn to retreat, but he can retreat to c7 instead of e7, penning the king nearer to the side and still making progress. This motif recurs at moves 13-14.

With like bishops, the result always becomes clear within a few moves and the play has little interest. There are three non-trivial positions in which the king loses with the move but can draw without it (Bd7/Bb5 against Ka8, Bd6/Bd4 against Kh5 or Kd1).

This ending appears to have been first examined by Leoncini and Magari in 1980.

Endings with **king against knight and a line-moving piece** are best regarded as drawn in the absence of a quick win for either side, but there are a large number of exceptions and it is possible to do little more than quote the bare statistics and give a few specimen lines. **King against queen and knight** offers 982 non-trivial wins for the king to move, and 293 non-trivial losses. Black's best plan, against a king in the corner a2/a1/b1, is to put his knight on d5, and if he can achieve this in safety he certainly wins. There are some other wins against a king on the edge (in particular, Nc5 against Kb1, although not infallible, is usually strong), but there is only one non-trivial win against a king away from the edge, Qd8/Nb6 against Kb2, and even then White's first move takes the king to the edge and Black's reply ...Nd5 keeps him there. A longest win for the pieces has Qb8/Nb6 against Ka1, with play

1	Ka1-a2 !	Qb8-c7 !	4	Kc1-b1 !	Qd6-e6
2	Ka2-b1 !	Qc7-d6 !	5	Kb1-a2	Qe6-c6
3	Kb1-c1	Nb6-d5 !	6	Ka2-a1 !	Qc6-c5 !

after which White must allow two sacrifices.

The king's strategy is to attack the queen, since a sacrifice will give the winning ending K v N, and if the knight is unable to intervene he can always achieve this. In practice, he has significant winning chances only if the knight is on or near the edge. There are no non-trivial wins at all against a knight on d5, and very few against a knight on d6 (the longest is Ke1 against Qg6/Nd6, play 1 Kf2 Qh7 2 Kf3 with a sacrifice on e4) or c6 (the longest is Kd1 against Qf6/Nc6, play similarly). A longest win overall has Kf1 against Qc8/Na7, with play

1	Kf1-g2 !	Qc8-d8 !	4	Kf2-e3	Qd8-g8 !
2	Kg2-g3 !	Qd8-a5 !	5	Ke3-f3 !	Qg8-d8
3	Kg3-f2 !	Qa5-d8 !	6	Kf3-f4 !	

and the queen is dominated. 5...Qh8 6 Kf4 comes to the same thing.

There are two non-trivial positions in which the king loses with the move but can draw without it (Qc8/Nd3 against Kh6, Qd8/Nb6 against Kb2) and six in which the same is true of the pieces (Qa8/Nb7 against Ke4, Qa8/Nh2 against Kd2, Qb8/Nb7 against Kb3 or Kd3, Qb8/Nc7 against Kf4, Qb8/Ng7 against Kd3). The first of these (Qc8/Nd3 against Kh6) was exploited in a study published by Fabrice Liardet in 1997.

All in all, this is arguably the most difficult of the three-piece endings.

King against rook and knight offers only 178 non-trivial wins for the king to move, all involving an immediate sacrifice threat which cannot be countered, and 1076 non-trivial losses. A longest has Rc8/Nb8 against Ka1, with play

1	Ka1-b2	Nb8-c6 !	7	Kb2-c1 !	Rh4-h3 !
2	Kb2-c1	Rc8-e8 !	8	Kc1-d1 !	Rh3-f3 !
3	Kc1-c2 !	Re8-e7	9	Kd1-c1 !	Rf3-e3 !
4	Kc2-b2	Nc6-e5 !	10	Kc1-b1 !	Re3-d3 !
5	Kb2-c1	Re7-h7	11	Kb1-a1 !	Rd3-c3 !
6	Kc1-b2 !	Rh7-h4			

followed by two sacrifices. However, such a win is possible only against a king already on or near the edge. There are no non-trivial wins at all against a king in the central 16 squares or on a square such as d2, and only three against a king on c2 (Nc6, Re8/e7/e6).

The ending was considered by Klüver in 1923, and more extensively by Fabrice Liardet in 1997.

King against bishop and knight offers 1714 non-trivial wins for the king to move, and 192 non-trivial losses. The latter figure is surprising, because neither bishop nor knight can normally win on its own, but if the king is trapped against the edge one piece may be able to sacrifice itself so as to produce an exceptional winning position for the other. A longest win has Bf1/Ng4 against Kb1, with play

1	Kb1-c1 !	Bf1-a6 !	6	Ka1-a2 !	Be2-b5 !
2	Kc1-b1 !	Ng4-f2	7	Ka2-b1 !	Ne4-d2 !
3	Kb1-b2 !	Ba6-f1 !	8	Kb1-a1 !	Nd2-b3 !
4	Kb2-b1	Nf2-e4 !	9	Ka1-b1	Nb3-a1
5	Kb1-a1 !	Bf1-e2			

followed after 10 KxS by 10...Ba4. The little flourish at moves 7 and 8 will be noted. Black can safely attack the king by 7...Kd2, but after 8 Ka1 the sacrifice on b1 will only draw. However, he can attack again by 8...Nb3, and now the sacrifice is decisive. Fabrice Liardet points out that the play can be extended by setting bBg2, Black to move, when the only winning move is ...Bf1.

The general defensive strategy for the pieces is to place the knight on a square of the same colour as the bishop. The king cannot now threaten to sacrifice himself on a square which is doubly guarded, and his only other option is to chase the bishop. A longest win has Ka2 against Nb7/Bd8, with play

1	Ka2-b3 !	Bd8-h4 !	5	Ke3-e4 !	Bd8-a5 !
2	Kb3-c3 !	Bh4-d8 !	6	Ke4-f4 !	Nb7-d8
3	Kc3-d3 !	Bd8-c7 !	7	Kf4-e3 !	
4	Kd3-e3 !	Bc7-d8 !			

and the bishop is dominated (or 6...Bb4 7 Ke5 with a sacrifice on d6). However, perhaps more instructive is Kh3 against Nd5/Bf7, when we have

1	Kh3-h4 !	Bf7-g8 !	4	Kg5-g4 !	Bd5-c4
2	Kh4-h5 !	Nd5-c7 !	5	Kg4-f5 !	
3	Kh5-g5 !	Bg8-d5 !			

with a sacrifice to follow. Here Black starts with his knight on a square of the right colour, but it obstructs his bishop and he has to move it away. This is an ending where the knight may be better out of the way in the corner rather than on the central square which it usually seeks; if it were on a8 here instead of on d5, the game would be drawn.

There are two non-trivial positions where the king loses with the move but can draw without it (Bd7/Ne5 against Ka7, Bd7/Nf7 against Ka8) and four where the same is true of the pieces (Bd6/Nb7 against Kc2, Bb8/Ne5 against Ka5, Bd8/Nh4 against Kd4, Bd6/Nd8 against Kd3).

King against two knights offers 2342 non-trivial wins for the king to move, 3024 draws, and 46 non-trivial losses. If the knights are separated, the king can hope to hunt one of them down, force it to sacrifice itself, and then win against the other; if the knights are close together but near to the edge of the board, the king may be able to manoeuvre just out of range and win by domination. However, if the knights can safely place themselves on adjacent or diagonally adjacent squares away from the edge they can always hold the draw.

A longest win is given by Kh3 against Nc8/Ng7, with play

1	Kh3-g4	Ng7-e8 !	5	Kf5-e4	Nb5-a3 !
2	Kg4-f3 !	Ne8-c7 !	6	Ke4-e5	Na3-b1 !
3	Kf3-f4 !	Nc8-a7 !	7	Ke5-d4	
4	Kf4-f5 !	Nc7-b5 !			

and a winning capture or sacrifice next move. The position of the knights after 3...Na7 is unfortunate (the knight at a7 being temporarily out of range, the king can attack e6 and drive the other knight away from c7), but if Black plays 3...Nb6 instead White will play 4 Ke4 and sacrifice on d5.

The non-trivial long wins for the knights all involve a king on the edge. A longest has Nc6/Ne4 against Kg8, with play 1 Kh7 (best) Ng5 2 Kh8 Nf7! (2...Nh7 loses) 3 K-- Nh8 4 KxN Ne5 and a win by domination.

There are four non-trivial positions where the king loses with the move but can draw without it (Nc6/Ne4 against Kg8, Nd6/Nc5 against Kh8, Nd6/Ne4 against Kh7, Nd5/Ne4 against Ka8) and eight where the same is true of the knights (Na8/Nb7 against Ke4, Nc8/Nc7 against Kf4 or Kc3, Nc8/Na6 against Ke4, Nd8/Nd7 against Kd3,

Nb7/Nf5 against Kb3, Nc7/Ng5 against Kc3, Nc7/Ng3 against Kg7).

Endings with king against king and a line-moving piece are best regarded as drawn in the absence of a trivial win for either side, but there are some exceptional positions. **King against king and queen** offers 261 non-trivial wins for the king to move, and 256 non-trivial losses. The most interesting of the wins involve a masked queen, and a longest features Kh3 against Ke4/Qa8: 1 Kg2 Kd5 2 Kf3 Kc6 3 Ke4 Kb7 4 Kd5 (Ke5 also works) and a sacrifice next move. Positions of this kind were exploited by Fabel and Klüver in 1947.

All the non-trivial wins for the pieces have the lone king on the edge or on a square equivalent to b2. If the lone king can reach any more central square in safety he can certainly hold the draw, and even on the edge there are many drawn positions. A longest win has Qa8/Ka4 against Ka1, with play

1	Ka1-b1 !	Qa8-f8	9	Ke1-d1	Qd6-f8
2	Kb1-c1 !	Ka4-b4 !	10	Kd1-c1 !	Qf8-e7
3	Kc1-d1 !	Kb4-c4 !	11	Kc1-b1 !	Kd4-c4 !
4	Kd1-e1 !	Qf8-a3 !	12	Kb1-c1 !	Qe7-e8
5	Ke1-d1 !	Qa3-a8 !	13	Kc1-b1 !	Qe8-d7
6	Kd1-e1 !	Kc4-d4 !	14	Kb1-a1 !	Kc4-b4
7	Ke1-f1 !	Qa8-a3 !	15	Ka1-b1 !	Qd7-d8
8	Kf1-e1	Qa3-d6 !	16	Kb1-a1 !	Qd8-c7

followed by two sacrifices.

There is one non-trivial position in which the king loses with the move but can draw without it (Kd6/Qc8 against Ka4).

King against king and rook, with the king to play, offers 260 non-trivial wins for the king (all involving a threat of immediate sacrifice which Black cannot counter) and 568 for the pieces. As with king and queen, all the non-trivial wins for the pieces have the lone king on the edge or on a square equivalent to b2, and a longest win has Ka5/Rh3 against Kb1: play

1	Kb1-a1 !	Rh3-g3	8	Ke1-f1 !	Kc4-d5 !
2	Ka1-b1 !	Ka5-b5	9	Kf1-e1 !	Kd5-d4 !
3	Kb1-c1 !	Kb5-c5	10	Ke1-f1 !	Rc3-d3 !
4	Kc1-d1 !	Kc5-d5	11	Kf1-g1 !	Kd4-e5 !
5	Kd1-e1 !	Rg3-b3 !	12	Kg1-f1	Ke5-e4 !
6	Ke1-d1 !	Kd5-c4 !	13	Kf1-g1 !	Rd3-e3 !
7	Kd1-e1 !	Rb3-c3 !	14	Kg1-h1 !	Re3-f3 !

followed by the usual two sacrifices.

This ending was considered by Leoncini and Magari in 1980.

King against king and bishop, with the king to play, puts matters the other way round; it offers 337 non-trivial wins for the lone king, but only four for the pieces. All the interesting wins for the lone king feature a masked bishop, a longest having Kh5 against Ba8/Kc6: 1 Kg4 Kd5 2 Kf3 Kc6 3 Ke4 (Kf4 also works) Kb7 4 Kd5 (or Ke5) and a sacrifice next move. This also was exploited by Fabel and Klüver in 1947.

The non-trivial wins for the pieces are all positions of immediate domination (Bd8 and Kd7/d5/b5/a5 against Ka8). Kd6 and Kc5 are not included on account of 1 Kb8 and 1 Ka7 respectively.

King against king and knight, with the king to move, offers 2040 non-trivial wins for the king but only 14 for the pieces. If the king and knight are together, even if they are cramped side by side on a8 and b8, the lone king can win only if it can threaten an unavoidable sacrifice (for example, by playing Kd6 against Na8/Kb8); otherwise, the pieces can hold the draw. If the pieces are separated, however, the lone king may be able to chase the opposing king, force it to sacrifice itself, and then win with K v N. A longest win occurs with Ke8 against Ke5/Na8: play 1 Kf7 (not 1 Ke7, when 1...Kd6 2 KxK Nc7 wins for Black) Kd4 2 Ke6 Kc3 3 Kd5 Kb2 4 Kc4 Ka1 5 Kb3 and so on.

The non-trivial wins for the king and knight all feature a dominated king in the corner.

Finally, king against two kings, with the lone king to move, offers only ten non-trivial wins for the lone king and six for the two kings, and each is very simple; for example, Kd5 wins against Kd8/Ke8 (1 Kd6/Ke6), and Ka8 loses against Kd8/Ka5.

[Correction to this last, noted 28 March 2007: the wins are typified by Ka8 against Kd8/Kd7 (1 Kb8).]

- Three-man pawnless endings in Losing Chess (July 1998) -

Appendix 1: raw computer statistics

- Men -	T	---	Wins	---	Draws	--	Losses	--	Max	Max	Loss	Loss	Draw		
W	-B-	M	1	2	3+	1	2+	1	2	3+	win	loss	/win	/draw	/win
Q v Q+Q	W	9610						6112							
Q v R+Q	W	18018						13314							
Q v R+R	W	8294						7428							
Q v B+Q	W	17569						13763							
Q v B+R	W	16479						14853							
Q v B+B	W	7796						7926							
Q v N+Q	W	19097						12235							
Q v N+R	W	17769						13563							
Q v N+B	W	17228		3				14100		1			1		
Q v N+N	W	8947		54	2			6698		21			3		21
Q v K+Q	W	18627						12705							
Q v K+R	W	17252						14080							
Q v K+B	W	16616		9			2	14696		7	2		3		3
Q v K+N	W	17886		105	22		19	13128		108	64		5	9	13
Q v K+K	W	8293		61	2		105	7140		42	79		3	8	4
R v Q+Q	W	11536						4186							
R v R+Q	W	22120						9212							
R v R+R	W	10612						5110							
R v B+Q	W	22027						9305							
R v B+R	W	21187						10145							
R v B+B	W	10476		32				5206		8					
R v N+Q	W	23720						7612							
R v N+R	W	22795						8537							
R v N+B	W	22404		302	19		24	8569		14			4		
R v N+N	W	11248		360	281		197	3614		2	20		8	12	1
R v K+Q	W	24682						6650							
R v K+R	W	23842						7490							
R v K+B	W	22829		575	54		112	7757		5			4		1
R v K+N	W	22078		939	554		1677	6084					11		
R v K+K	W	9956		735	85		2195	2751					4		3
B v Q+Q	W	13031						2690		1					
B v R+Q	W	25610		3				5716		3					
B v R+R	W	12678		3				3039		2					
B v B+Q	W	24785		19		2240	244	4044							10
B v B+R	W	24197				2240	513	4380		2					
B v B+B	W	8090				2240	4000	1392							
B v N+Q	W	26938		216	12		1	4140		25			4		1
B v N+R	W	26421		388	4			4476		41	2		3	3	2
B v N+B	W	19019		107	7	2240	7153	2806					3		
B v N+N	W	11022		2384	771			1535		10			4		9
B v K+Q	W	24888		15	6	1597	160	4638		24	4		3	4	4
B v K+R	W	24344		19	1	1597	300	5010		38	23		3	8	1
B v K+B	W	17798		30	5	3871	6284	3344					3		
B v K+N	W	20614		1676	2889	1528	1107	3469		27	22		7	6	10
B v K+K	W	9502		172	7	1584	2325	2065		6	61		3	7	1
N v Q+Q	W	13447		113				1580		582					
N v R+Q	W	25490		243				3490		2028	81		4		
N v R+R	W	11847		241				1946		1370	318		3		
N v B+Q	W	23683		329				3674		3515	131		4		
N v B+R	W	20528		293				4004		4881	1626		4		13
N v B+B	W	8199		422				2126		3909	1066		5		1
N v N+Q	W	24014		1085	24			2715		1707	1787		4	5	3
N v N+R	W	20783		1191	6			3106		1782	4464		3	5	3
N v N+B	W	15768		4295	188		507	3293		2652	4629		6	12	40
N v N+N	W	7795		2174	1116		3062	1203		372			11	10	362
N v K+Q	W	23598		106	26			3727		3555	320		4	5	
N v K+R	W	20713		374	54			4081		3271	2839		4	4	
N v K+B	W	16296		269				4305		5560	4902			6	8
N v K+N	W	16022		1401	506		57	3370		1174	8802		5	13	82
N v K+K	W	8197		192	22			2193		732	4386		3	9	

(continued)

- Three-man pawnless endings in Losing Chess (July 1998) -

Appendix 1: raw computer statistics (continued)

- Men - W -B-	T M	Wins			Draws		Losses			Max win	Max loss	Loss /win	Loss /draw	Draw /win
		1	2	3+	1	2+	1	2	3+					
K v Q+Q	W	13329	59				2051	86	197		5	1		
K v R+Q	W	24804	197			8	4508	224	1591		7	1	1	
K v R+R	W	10210	270				2497	56	2689		7			
K v B+Q	W	24646	170	5	1120	292	3525	84	1490	3	17	1	2	
K v B+R	W	20321	45		1120	37	3931	146	5732		14			
K v B+B	W	10359	237	11	1120	1309	1556	13	1117	3	23		3	
K v N+Q	W	25440	265	717		1575	3042	138	155	7	8		2	6
K v N+R	W	19692	178			6869	3517	237	839		13			
K v N+B	W	19883	1120	594	1120	5808	2615	76	116	8	10		2	4
K v N+N	W	9170	612	1730		3024	1140	41	5	8	4		4	8
K v K+Q	W	22944	195	66	2365	2991	2515	8	248	5	18			1
K v K+R	W	15735	260		2365	9462	2942	12	556		16			
K v K+B	W	16362	208	129	3485	9140	2004	4		5				
K v K+N	W	14088	493	1547	2365	11283	1542	14		6				
K v K+K	W	4437	10		2365	8395	509	6						

Appendix 2: extracts from computer analysis

Extract from analysis of Q v N+B, White to move

List of non-trivial losses (wins with opponent to move)

Nc7/Bb8 v Qh4

Extract from analysis of Q v K+B, White to move

List of non-trivial losses (wins with opponent to move)

Kc7/Bd8 v Qa2

Kd7/Bc8 v Qf2

Kd7/Be8 v Qb2

Extract from analysis of Q v K+N, White to move

List of non-trivial losses (draws with opponent to move)

Kd8/Na3 v Qh6

Kc6/Nh7 v Qe1

List of non-trivial draws (wins with opponent to move)

Kb8/Na3 v Qf6

Kc8/Nh7 v Qe2

Kd8/Nh7 v Qb3

Kc7/Nh8 v Qe2

Kc7/Nh7 v Qa2

Extract from analysis of Q v K+K, White to move

List of non-trivial losses (wins with opponent to move)

Kc8/Kd8 v Qh6

Kc8/Ke1 v Qg6, a3

Kd8/Ka5 v Qf3

List of non-trivial losses (draws with opponent to move)

Ka8/Kd8 v Qh6

Kb8/Ka4 v Qf6

Kb8/Kh2 v Qf6

Kc8/Kd8 v Qa1

Kd8/Kc7 v Qa2

Kd8/Ke7 v Qg2

List of non-trivial draws (wins with opponent to move)

Ka8/Kh6 v Qc4

Kb8/Kc8 v Qg6

Kb8/Kd1 v Qf6

Appendix 2: extracts from computer analysis (continued)

Extract from analysis of R v N+N, White to move

List of non-trivial losses (wins with opponent to move)

Nc7/Ne6 v Rh3

List of non-trivial losses (draws with opponent to move)

Nd7/Ne5 v Ra2

Nd7/Nc4 v Rg1

Nd7/Ng4 v Ra1

Nc6/Nd5 v Rg2

Nd5/Ne4 v Ra8

List of non-trivial draws (wins with opponent to move)

Nc8/Nb5 v Rf2

Nd8/Ne7 v Ra4

Nd8/Nc6 v Rg3

Nd8/Na5 v Rg2

Nb7/Ne7 v Rh4

Nb7/Nc6 v Rf3

Nc7/Nd7 v Rg4

Extract from analysis of R v K+B, White to move

List of non-trivial losses (wins with opponent to move)

Kc6/Be4 v Ra8

List of non-trivial draws (wins with opponent to move)

Kd6/Bf4 v Ra8

Extract from analysis of R v K+K, White to move

List of non-trivial draws (wins with opponent to move)

Ka8/Kd8 v Rf6

Ka8/Kb7 v Rd5

Kb8/Ka7 v Rd5

Appendix 2: extracts from computer analysis (continued)

Extract from analysis of B v B+Q, White to move

List of non-trivial draws (wins with opponent to move)

Bc8/Qa8 v Bg5
Bd8/Qb8 v Bh5
Bc7/Qa7 v Bg6,g4
Bd7/Qb7 v Bh4
Bd7/Qh7 v Bb6
Bc6/Qc8 v Bf2
Bd6/Qd8 v Bc2
Bd6/Qb6 v Bh3
Bd5/Qh5 v Bb6

Extract from analysis of B v N+Q, White to move

List of non-trivial losses (wins with opponent to move)

Nc7/Qa7 v Bh7

Extract from analysis of B v N+R, White to move

List of non-trivial losses (wins with opponent to move)

Nd8/Ra8 v Bh7
Nc7/Ra7 v Bh7

Extract from analysis of B v K+Q, White to move

List of non-trivial losses (draws with opponent to move)

Kd7/Qa7 v Bh7
Kd6/Qd7 v Bd2
Kd5/Qd7 v Bc1,e1

List of non-trivial draws (wins with opponent to move)

Kc8/Qb8 v Bh5
Kc7/Qa7 v Bg6
Kd6/Qd8 v Bc2

Extract from analysis of B v K+R, White to move

List of non-trivial losses (draws with opponent to move)

Kd7/Ra7 v Bh7

Appendix 2: extracts from computer analysis (continued)

Extract from analysis of B v K+N, White to move

List of non-trivial losses (draws with opponent to move)

Kd8/Nd7 v Bc1,e1

Kc7/Nd7 v Bd2

Kd6/Nd7 v Be1

Kd6/Nf7 v Be1

Kd5/Ne8 v Bd2

Kd5/Nf8 v Bd1

Kd5/Nd7 v Bd2

Kd5/Nh5 v Ba5

Kd5/Ng4 v Ba5

List of non-trivial draws (wins with opponent to move)

Ka8/Nf5 v Ba5

Ka8/Ng4 v Ba5

Ka8/Nh3 v Ba5

Kc8/Ne5 v Bc5

Kd8/Nf5 v Bd5

Extract from analysis of B v K+K, White to move

List of non-trivial draws (wins with opponent to move)

Kd8/Ka5 v Bd5

Extract from analysis of N v B+R, White to move

List of non-trivial losses (wins with opponent to move)

Ba8/Rb7 v Nf4,h4,g3,h2

Bb8/Rc7 v Ng4,f3,h3,g2,f1,h1

Bc8/Rd7 v Ng3,h2,g1

Extract from analysis of N v B+B, White to move

List of non-trivial losses (wins with opponent to move)

Bc8/Bh3 v Nc3

Extract from analysis of N v N+Q, White to move

List of non-trivial losses (wins with opponent to move)

Nc6/Qa8 v Nf4

Nd6/Qb8 v Ng4

Nd6/Qa6 v Ng4

Extract from analysis of N v N+R, White to move

List of non-trivial losses (wins with opponent to move)

Na8/Rc7 v Nf4

Nb8/Ra8 v Ne6,d5

Appendix 2: extracts from computer analysis (continued)

Extract from analysis of N v K+B, White to move

List of non-trivial losses (wins with opponent to move)

Kb7/Ba8 v Nf6, f4

Kc7/Bb8 v Ng4, d3, f3

Kd7/Bc8 v Ne3, g3

Kd7/Be8 v Nc3

List of non-trivial losses (draws with opponent to move)

Kb8/Nh7 v Ne5

Kb8/Nh5 v Ne5

Kd8/Na8 v Ng5

Kb7/Nh6 v Ne4

List of non-trivial draws (wins with opponent to move)

Kb8/Nb1 v Nf8

Kb8/Nd1 v Nf8

Kd8/Na8 v Nf4

Extract from analysis of K v Q+Q, White to move

List of non-trivial losses (wins with opponent to move)

Qb8/Qf8 v Kd3

Extract from analysis of K v R+Q, White to move

List of non-trivial losses (wins with opponent to move)

Rd7/Qc8 v Kf5

List of non-trivial losses (draws with opponent to move)

Rc6/Qa8 v Ke4

Extract from analysis of K v B+Q, White to move

List of non-trivial losses (wins with opponent to move)

Bd7/Qd8 v Kd4

List of non-trivial losses (draws with opponent to move)

Bd8/Qb8 v Kd2

Bd7/Qg8 v Ke3

Extract from analysis of K v B+B, White to move

List of non-trivial losses (draws with opponent to move)

Bd7/Bb5 v Ka8

Bd6/Bd4 v Kh5, d1

Appendix 2: extracts from computer analysis (continued)

Extract from analysis of K v N+Q, White to move

List of non-trivial losses (draws with opponent to move)

Nc7/Qa5 v Kg7

Nd6/Qc1 v Kh3

List of non-trivial draws (wins with opponent to move)

Nb8/Qh1 v Kb4

Nb7/Qa8 v Ke4

Nb7/Qb8 v Kb3, d3

Nb7/Qg8 v Ke3

Nc7/Qb8 v Kf4

Extract from analysis of K v N+B, White to move

List of non-trivial losses (draws with opponent to move)

Nd5/Be7 v Kh7

Nc7/Be7 v Kh8

List of non-trivial draws (wins with opponent to move)

Nb7/Bd6 v Kc2

Nd8/Bd6 v Kd3

Nd8/Bh4 v Kd4

Nd5/Bg8 v Kh5

Extract from analysis of K v N+N, White to move

List of non-trivial losses (draws with opponent to move)

Nc6/Ne4 v Kg8

Nd6/Nc5 v Kh8

Nd6/Ne4 v Kh7

Nd5/Ne4 v Ka8

List of non-trivial draws (wins with opponent to move)

Na8/Nb7 v Ke4

Nc8/Nc7 v Kf4, c3

Nc8/Na6 v Ke4

Nd8/Nd7 v Kd3

Nb7/Nf5 v Kb3

Nc7/Ng5 v Kc3

Nc7/Ng3 v Kg7

Extract from analysis of K v K+Q, White to move

List of non-trivial losses (draws with opponent to move)

Kd6/Qc8 v Ka4