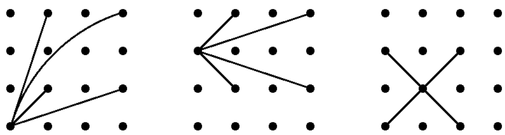
A class of complementary four-move leaper tours with horizontal or rotational symmetry

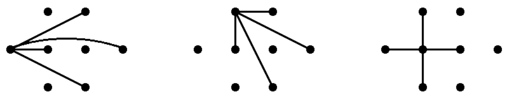
John Beasley, 15 March 2023

A **four-move leaper** is a piece which, from any point on a square or other board, has precisely four moves available, and a pair of complete tours of the board by a four-move leaper is called **complementary** if at each point one of the tours uses the two moves that the other tour does not. This paper looks at some pairs of complementary four-move leaper tours which have horizontal or 180-degree rotational symmetry.

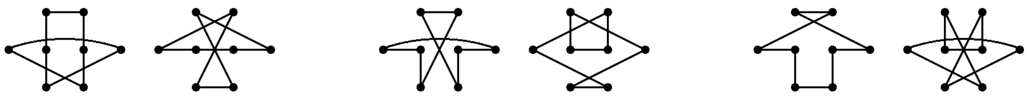
On a 4×4 board, the combined (1,1)/(1,3)/(3,3) leaper is a four-move leaper:



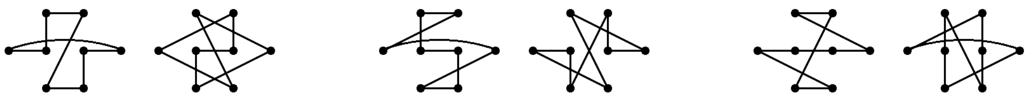
Such a leaper cannot tour the whole board (if we imagine the points labelled alternately odd and even, chessboard style, each leap takes it to a point with the same label), but it can visit all the points with the same label as its starting point. To reduce clutter, we shall omit from our future diagrams the points which it cannot visit, and we shall rotate the board by 45 degrees so that the previous diagonal lines become horizontal and vertical. In effect, we shall be examining the tours of a combined (0,1)/(0,3)/(1,2) leaper on an 8-point lozenge-shaped board:



There are six tours of this board by a combined (0,1)/(0,3)/(1,2) leaper with horizontal symmetry, and they form three complementary pairs:



There are also six tours with 180-degree rotational symmetry, and again they form three complementary pairs:



Other pairs of complementary tours exist and indeed every tour can be shown to have a complementary tour, but no tour can be converted into its complementary tour by reflection or 180-degree rotation (if a tour includes a leap between the two inside points, the reflected or rotated tour will also include a leap between these two points, and so cannot be the complementary tour).