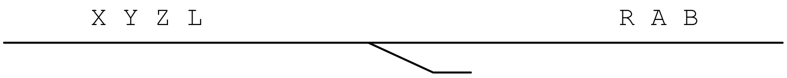
Variations on a railway shunting problem

JDB, 26 November 2015, solutions improved 29 November / 15 December

There are a number of classic railway shunting problems on the general theme that two trains have to pass each other on a single line and the passing loop is too short to allow one to pull up while the other goes through. Several are in chapter 11 of Edward Hordern's 1986 book *Sliding Block Puzzles*, and there are further references in section 5.P.1 of David Singmaster's *Sources in Recreational Mathematics*. I haven't seen all of these, so what follows may be original only in minor cosmetic detail, but if this is the case please will some reader tell me.

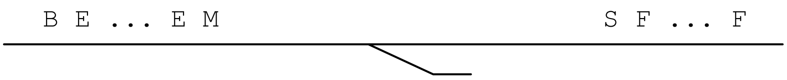
I take as my starting point a version given by Dudeney (problem 374 in Martin Gardner's 1967 edition *536 Puzzles and Curious Problems*). This has an engine and three coaches meeting an engine and two coaches on a single line, with a dead-end siding only large enough to accommodate one engine or one coach :



The engines are L and R, and the task is to get the trains past each other. As an additional twist, the problem is to be solved in as few moves as possible, but this is an aspect of the matter which we shall not consider here.

It is convenient to regard the tracks as if they were streams flowing from right to left and joining, so "upstream" is to the right, and with this notation Dudeney's solution is as follows. R on its own comes down past the points and goes into the siding; L pulls XYZ up past the points and couples on to A; R emerges from the siding and goes a little way down the line; L comes down past the points pushing XYZ and pulling A (R had to go far enough for this to be possible), parks A in the siding, pulls XYZ back up past the points, and couples on to B; R picks up A from the siding and takes it down the line; L brings XYZB down past the points, parks B in the siding, and takes XYZ on their way; R pushes A towards the siding, picks up B, and departs.

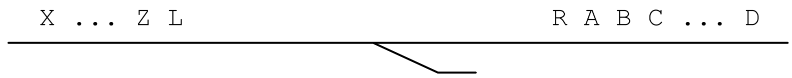
This solution generalizes to trains of any length, but it involves pushing as well as pulling, and there are obvious objections to pushing other than very short trains. It therefore occurred to me to try and solve the problem for trains of arbitrary length with the additional condition that not more than one coach be pushed at a time (each engine is assumed to be strong enough to *pull* as many coaches as required). This makes little difference as far as Dudeney's three-plus-two formulation is concerned, since the only pushes are of XYZ by L, which can be replaced by pulls by R, and of A towards the siding at the end, which is a push of one carriage only. However, it no longer works if we allow the right-hand train to consist of more than two coaches. The trains can be got past each other easily enough (R goes into the siding as before, L pulls X...Z up past the points, R emerges from the siding and pulls X...Z down past the points, L couples on to A, parks it in the siding, picks up X...Z, and pulls them up past the points, R picks up A from the siding, pushes it up to join X, and pulls AX...Z down past the points, L parks B in the siding and then pulls AX...Z up past the points, R picks up B from the siding and pulls BAX...Z down past the points, and so on), but the order of the coaches in the right-hand train is reversed in the process. Can we get the trains past each other without altering the order of the coaches?



In another variation, which is perhaps more likely to be original, we are on a branch line leading from the main line to a mine, a main-line locomotive M is bringing up a train of empty wagons plus a brake van, the mine shunter S is bringing up a string of full wagons, and we wish to interchange these, forming a train of full wagons plus the brake van ready to go to the main line and putting the shunter at the head of a string of empty wagons for the mine. The siding can hold a single wagon, or the brake van, or the mine shunter, but not the main-line locomotive (though this can push a vehicle into the siding or pull one out), and only a single wagon or the brake van can be pushed at a time though each engine is strong enough to pull as many vehicles as required. Can it be done?

Solutions

Both problems can be solved. In our original problem as generalized, we have

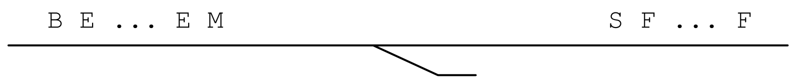


and all that is necessary is to reverse the order of A...D before applying the manoeuvre to get the trains past each other. This can be done as follows.

- L stops far enough short of the points to allow RA...D to be brought down without fouling them, and detaches. R detaches and goes into the siding, L pulls A...D below the points, R emerges and pulls B...D above the points leaving A with L.
- L pushes A into the siding and pulls B...D below the points, R extracts A from the siding and pushes it up to where D used to be. We now have LB...D below the points and RA above them.
- R pulls C...D above the points leaving B with L, L pushes B into the siding and pulls C...D below the points, R extracts B from the siding and pushes it up to join A, giving LC...D below the points and RBA above them. This is repeated as often as necessary, each of C...D in turn being parked in the siding by L and then pushed up to join the upper string by R, eventually giving L alone below the points and RD...A above them.

This has reversed the order of coaches in the right-hand train, and we can now manoeuvre the trains past each other as before. By the end, R and each of A...D have visited the siding twice, but L and X...Z have remained on the main line and have not visited it at all. I cannot at the moment see how to reduce the total number of visits.

In the mine problem



we aren't worried about the order of wagons within strings, and a solution is as follows.

- S stops far enough short of the points to allow the main-line train to pull up to it without fouling them, M stops far enough short to allow S to pull up to it.
- S detaches and goes into the siding, M pulls BE...E above the points, and S emerges. We now have S alone below the points and BE...EM / F...F above them (M need not connect to F...F).
- S detaches B, pushes it into the siding, and pulls E...E below the points, M extracts B from the siding and pushes it up to F...F (it need not connect to them, but it must go far enough for ME...E eventually to be fitted in below without fouling the points). This gives SE...E below the points and MB / F...F above them.
- M pulls all but one of E...E above the points leaving the lowest E with S, S parks this E in the siding and pulls the remaining E...E below the points, M extracts the E from the siding and pushes it up to B (it need not connect to B). This is repeated for each of the remaining E...E in turn, the lowest E at each stage being parked in the siding by S and then pushed up to join the upper string by M (now the E being moved does need to connect to the upper string), eventually giving S alone below the points and ME...E / B / F...F above them.
- S goes into the siding, M pulls E...E below the points, S emerges and couples to B. We now have ME...E below the points and SB / F...F above them.
- S puts B into the siding and pulls E...E above the points, M extracts B from the siding and pushes it up to join E...E. We now have nothing below the points and MBE...ES / F...F above them.
- M pulls BE...E below the points, S puts the lowest F into the siding and pulls BE...E above the points, M extracts the F from the siding and pushes it up to join B. This is repeated for each of F...F in turn, the lowest F at each stage being parked in the siding by S and then pushed on to the bottom of the string by M, eventually giving MF...FBE...ES above the points.
- The trains divide above the brake van and go their separate ways.

Each wagon visits the siding once, and the brake van and the mine shunter visit it twice.