

A few lightweight puzzles and curiosities

JDB, most recent addition 12 April 2016, note added to solution 7 on 12 May

A few lightweight puzzles and curiosities which, like **8.1** and **8.2** in *51 flights*, are unlikely to be original but which I haven't seen attributed to anyone else.

1 Given 6 blue balls, 5 green, 4 orange, 3 pink, 2 red, and one yellow, divide them into seven groups of three such that (a) no two groups are the same and (b) no two balls in any group are of the same colour.

2 A darts cheat equips his darts with a GPS-based navigation and guidance system which enables him to throw a dart into any chosen sector of the board. Unfortunately it makes the darts so fat that no two can be thrown into the same sector at one turn. Even with this restriction, can he throw a nine-dart 501?

(The dart-board has sectors single-1 to single-20, double-1 to double-20, treble-1 to treble-20, inner (value 25), and bull (value 50, and counting as a double). A turn consists of three darts, hence the scorer's cry of "One hundred and EIGH-ty!!!" when a player hits three treble-20s, and a player must reach his 501 by throwing a double. So the task is to make 501 from nine of these possible scores, none being used more than three times and at least one being a double.)

3 This problem is set in the days when ties on points in football leagues were resolved by goal average and not by goal difference. Stoke City go into the last day of the season top of the League (happy thought!), Crewe Alexandra and Port Vale are one point behind, and nobody else matters. Stoke have scored 67 goals and conceded 54, Crewe have scored 60 and conceded 49, Port Vale have scored 71 and conceded 58. Crewe and Port Vale are due to play each other in the last match. If Stoke lose their last match 1-0 (biased ref) and Crewe and Port Vale draw, who will be champions?

4 The dominoes in a standard 28-domino set are two inches long, one inch across, and a third of an inch thick. Place seven dominoes on edge so that the remaining 21 can be laid on them to make a flat roof.

(In theory, three dominoes can be balanced on one by putting them crosswise along a diagonal of the supporting domino, since a domino is only one inch wide and the diagonal of a two-inch domino even standing on edge is longer than two inches. However, if the thickness of the two-inch domino is only a third of an inch, its diagonal when standing on edge is less than 2.03 inches, so at least one centre point of an upper domino must be less than 0.015 inches from the corner of the supporting domino. I have been unable to get this to stand up with any set of real dominos which I have encountered. The solution sought here has all the dominoes oriented strictly north-south or east-west, none being slantwise, and works with typical dominoes as found in the shops as long as they are not too badly worn.)

5 A wallpaper manufacturer decides to base his repetitive tessellation on a standard set of 28 dominoes, arranging them so that the blanks, ones, and so on form groups of eight connected squares when the pattern is replicated over the plane. Can it be done? The eight squares of a group must be connected edge-wise, mere corner-wise connection not being enough.

6 What simple race game, played with ordinary equipment, most naturally uses a course of 67 squares?

7 What counts 0, 1, 2, 3, 4, 9, 8, 7, 8, 9?

Solutions on following pages.

Solutions

- 1 Given 6 blue balls, 5 green, 4 orange, 3 pink, 2 red, and one yellow, divide them into seven groups of three such that (a) no two groups are the same and (b) no two balls in any group are of the same colour.

The only way is

B	G	O	-	-	-
B	G	-	P	-	-
B	G	-	-	R	-
B	G	-	-	-	Y
B	-	O	P	-	-
B	-	O	-	R	-
-	G	O	P	-	-
6	5	4	3	2	1

- 2 A darts cheat equips his darts with a GPS-based navigation and guidance system which enables him to throw a dart into any chosen sector of the board. Unfortunately it makes the darts so fat that no two can be thrown into the same sector at one turn. Even with this restriction, can he throw a nine-dart 501?

501 is a multiple of 3, as is every treble, so let us first suppose that the final double also is a multiple of 3. The highest such double is double-18, which has value 36 and leaves 465 to be thrown by the remaining eight darts. Three treble-20s give us 180, three treble-19s give 171, two treble-18s give 108: no, only 459, not enough.

All right, so the final double cannot be a multiple of 3. Suppose it is a bull (value 50). This leaves remainder 2 on division by 3, so it must be balanced either by a single non-treble leaving remainder 1 or by two more non-trebles each leaving remainder 2. Suppose the former. The largest such non-treble is double-20, which added to the bull gives 90 and leaves 411 to be made by the remaining seven darts. Three treble-20s give 180, three treble-19s give 171, one treble-18 gives 54: only 405, again not enough.

So we need two more non-trebles each leaving remainder 2. Let them both be bulls, then we need 351 from the remaining six darts: ah, three treble-20s at 180 and three treble-19s at 171.

So the solution is to throw treble-20, treble-19, bull at each turn, the last dart in the final turn being a bull.

- 3 In the days when ties on points in football leagues were resolved by goal average, Stoke City go into the last day top of the League, Crewe Alexandra and Port Vale are one point behind, and nobody else matters. Stoke have scored 67 goals and conceded 54, Crewe have scored 60 and conceded 49, Port Vale have scored 71 and conceded 58. Crewe and Port Vale are due to play each other in the last match. If Stoke lose their last match 1-0 and Crewe and Port Vale draw, who will be champions?

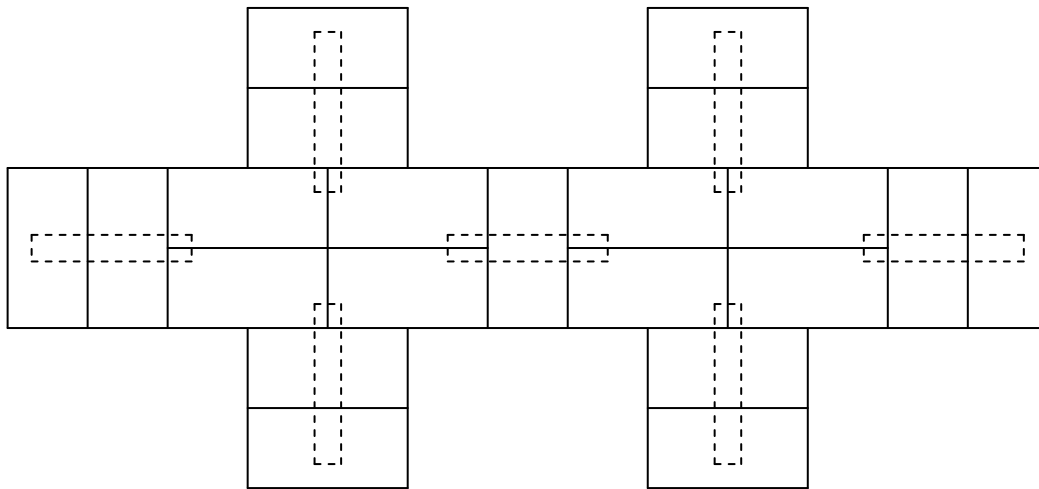
This is a curiosity rather than a puzzle, because the solution is merely a matter of arithmetic. After their 1-0 defeat, Stoke have scored 67 goals and conceded 55, giving them a goal average of 1.2182. As for Crewe and Port Vale, we have

Score	Crewe			Port Vale			Stoke
	F	A	Average	F	A	Average	Average
0-0	60	49	<u>1.2245</u>	71	58	1.2241	1.2182
1-1	61	50	1.2200	72	59	<u>1.2203</u>	1.2182
2-2	62	51	1.2157	73	60	1.2167	<u>1.2182</u>
more			less			less	<u>1.2182</u>

So if Crewe and Port Vale draw 0-0, Crewe are champions; if they draw 1-1, Port Vale are champions; and if they draw by 2-2 or anything greater, Stoke are champions.

- 4 The dominoes in a standard 28-domino set are two inches long, one inch across, and a third of an inch thick. Place seven dominoes on edge so that the remaining 21 can be laid on them to make a flat roof.

A solution which works with typical shop dominoes, at least when reasonably new, is shown below.



Either or both of the 4x2 rectangles at the crossing points can be set north-south instead of east-west.

When set up, it looks like a futuristic bus station.

- 5 A wallpaper manufacturer decides to base his repetitive tessellation on a standard set of 28 dominoes, arranging them so that the blanks, ones, and so on form groups of eight connected squares when the pattern is replicated over the plane. Can it be done? The eight squares of a group must be connected edge-wise, mere corner-wise connection not being enough

Indeed it can, and it can even be done with all the twos pointing in the same direction, likewise the threes, and likewise the sixes. Two specimen solutions with this property are on the following page.

- 6 What simple race game, played with ordinary equipment, most naturally uses a course of 67 squares?

There are no doubt several possible answers, but surely the simplest is a pursuit race around a circular track played with two ordinary cubic dice. A number of players from two to six start at various points around the track, a player caught or overtaken drops out, and the last survivor wins. If the players move simultaneously, the chances can be equalized by placing them evenly around the track, and a track of length 60 is the shortest that allows this to be done for any number of players from two to six inclusive. If the players throw in turn, the chances can be most nearly equalized by putting an extra seven squares in front of one of them, letting him throw first, letting the player immediately behind him throw second, and so on.

(I say “most nearly equalized” because it seems unlikely that the equalization is exact. I haven’t asked my computer to work out the probabilities for a track of length 67, but if we restrict the game to two players, replace the dice by coins giving a value of 0 or 1, and let the circuit be of five squares, we don’t need a computer, and we find (I hope) that the probability that the player to move wins is 232/249, 184/249, 124/249, or 68/249 according as he has 0, 1, 2, or 3 empty squares in front of him. The case corresponding to ours is that of two empty squares, where we see that the probability is not quite 1/2 though it differs by barely a fifth of one per cent. A similar imbalance can be expected to apply to games with cubic dice, though it presumably becomes ever smaller as the length of the track increases.)

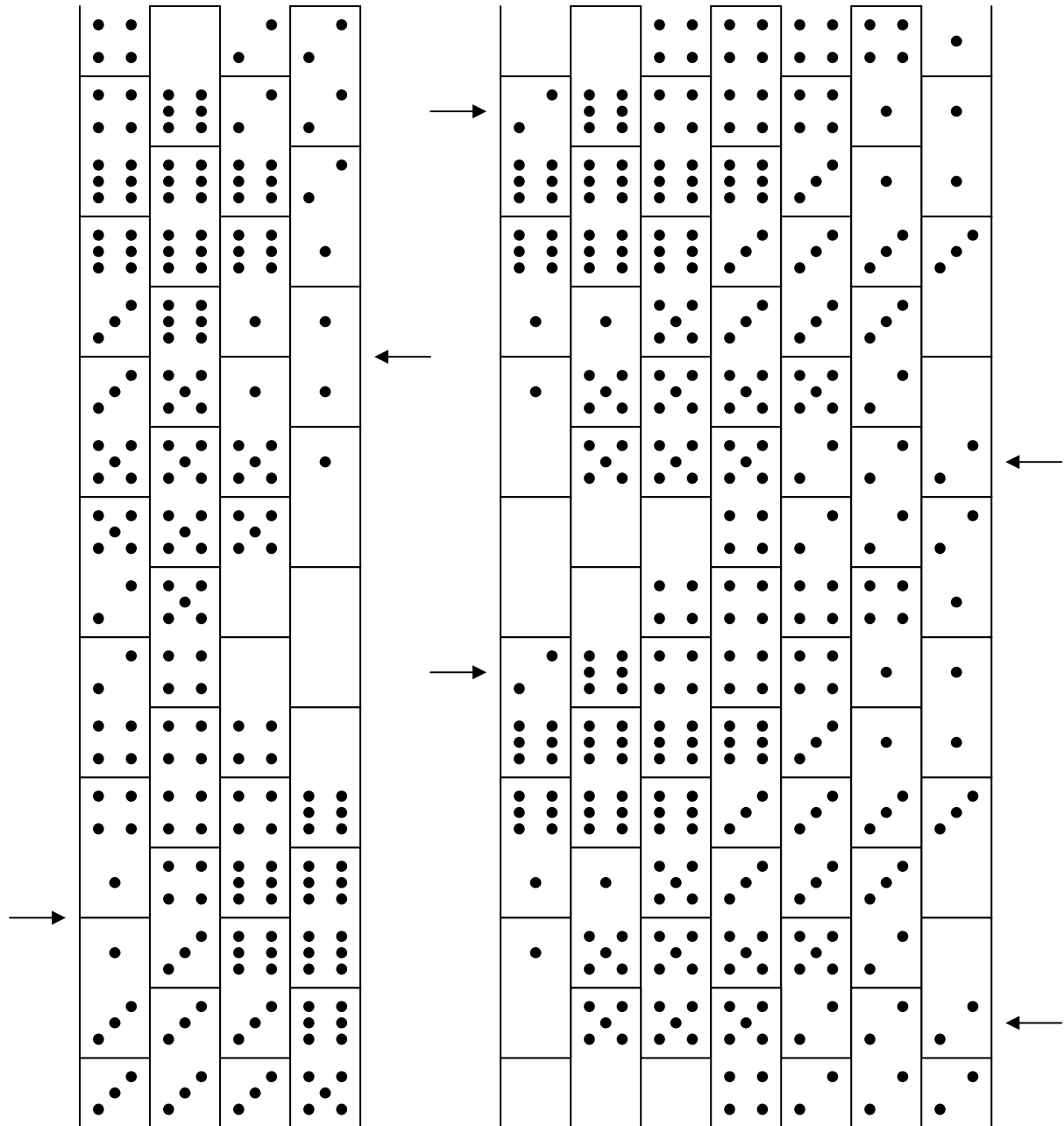
- 7 What counts 0, 1, 2, 3, 4, 9, 8, 7, 8, 9?

A faulty seven-segment display in which the upper right-hand segment is permanently “on”. I saw one of the station clocks at Haywards Heath doing just this on a day around 1990. Chris Maslanka was sufficiently amused when I sent it to him some twenty years later to have used it as one of his Saturday “Pyrgic Puzzles” in *The Guardian*.

(David Singmaster, in *Problems for Metagrobologists* (2016), has another version, also from real life, where a digital clock shows 6:59 and a little later 5:01. This time the faulty segment is permanently “off”.)

Specimen wallpaper domino tessellations

If we set all the dominoes vertically, or all horizontally, we shall ensure that all the twos, threes, and sixes point the same way. Furthermore, wallpaper comes in vertical rolls which have to be hung side by side, and if we choose the vertical option the two halves of a domino will never have to be matched up across a join. Two specimen solutions are shown below. When two rolls are hung side by side to extend the pattern horizontally, they must be placed so that the arrowed points match up, one or more rows being discarded from one of them as necessary.



The left-hand solution, with the eight occurrences of each value symmetrically arranged, was drawn to my attention by Noam Elkies. Only part of a roll is shown, the pattern repeating after 28 rows. A roll may contain any even number of columns.

The right-hand solution lacks this symmetry but is more compact, so that the matching-up of neighbouring rolls is likely to involve the discarding of fewer rows. The number of columns in a roll must be a multiple of seven, but the pattern repeats after eight rows instead of 28.

I don't know if a wallpaper using either of these designs has ever been put into production.