Two notes on magic knight's tours

John Beasley, 6 March 2012

Some eighteen months or so ago, George Bell mentioned to me that *The College Mathematical Journal* was planning a special issue in tribute to the late Martin Gardner. Since Martin had written on magic knight's tours in one of his columns in the 1970s, it occurred to me that the magazine's readers might be interested in a survey of subsequent progress, and the editor agreed. With one possible exception, my role in this field has merely been one of a reporter, so I was not necessarily the most appropriate person to write the survey, but I had a high regard for Martin and wanted to put my name on the list of contributors.

A couple of points in this paper might bear expansion. Be it noted that I am adopting the definition of "magic" used in the knight's tour literature: a tour is "magic" if the numbers in each row and column add to the same total, any diagonal or other properties being a bonus.

1. George Jelliss's proof that there can be no magic knight's tour on a square of side 4*n*+2

As those who have looked at George's "Knight's Tour Notes" on his site www.mayhematics.com will already have seen, what George actually proved was the stronger result that there could be no magic knight's tour on a rectangle of sides 4*m*+2 and 4*n*+2. In presenting his proof simplified for a square only, I was aware that I was selling him short, but there was no other mention of magic rectangles in the paper and I didn't want to introduce them just in this one place. However, let me acknowledge the partial presentation here.

And since the only property of a knight's tour on a checkerboard that is used in the paper is that all the odd numbers fall on squares of one colour and all the even numbers on squares of the other, the proof applies also to tours of any piece whose moves on a checkerboard take it to light and dark squares alternately.

2. The diagonal properties of Jaenisch's 1859 tour

I first drew attention to the diagonal properties of Jaenisch's 1859 tour, Figure 1b in the paper (the principal odd diagonal and each parallel odd broken diagonal add to 256, and the principal even diagonal and each parallel even broken diagonal to 264) when presenting it in *Variant Chess* 57 in 2008:

256	256		256		256				
	↘		↘		↘		↘		
		27	30	51	40	53	2	15	42 ← 260
		50	39	28	31	14	41	54	3 ← 260
	↗	29	26	37	52	**1**	56	43	16 ← 260
264		38	49	**32**	25	44	13	4	55 ← 260
	↗	23	36	45	12	57	**64**	17	6 ← 260
264		48	11	24	**33**	20	5	58	61 ← 260
	↗	35	22	9	46	63	60	7	18 ← 260
264		10	47	34	21	8	19	62	59 ← 260
	↗	↑	↑	↑	↑	↑	↑	↑	↑
264	260		260		260		260		
		260		260		260		260	

I wrote then that I could not believe that this elegant property had not been noticed before, but that I was not aware of any evidence to the contrary There is nothing in Jaenisch's 1859 *Chess Monthly* article to suggest that he was aware of it, nor could I find anything in his 1862 book *Traité des Applications de l'Analyse Mathématique au Jeu des Échecs* (this book is not the easiest of reading for somebody whose French is no more than passable, but I went through the relevant volume twice without success). There is no mention in Murray's unpublished 1951 monograph, from which I infer that there is no mention in any of the sources he used, I could find no mention in any of the other Murray papers preserved in the Bodleian Library, and there is no mention in George Jelliss's catalogue in *Chessics* 26. I invited readers of *VC* 57 to tell me if anyone had previously drawn attention to it, but nobody responded, and a similar appeal in the *CMJ* paper has likewise failed as yet to prompt a response (the magazine has now been out for at least six weeks).

Well, on the strength of this negative evidence I am perhaps willing to accept that I may have been the first to draw attention to this elegant property in print, but I still cannot believe that I was the first to have noticed it. It does indeeed make this "the most perfect knight's tour" on the 8 x 8 board.