# The cosmetics of puzzles

John Beasley, 3 November 2016, revisions to presentation 10-17 November[*]

In issues 13 and 14 of *Chessics* (1982), George Jelliss gave an exposition on a chessboard of hypercubes in up to six dimensions, and we subsequently composed problems based on them. I later quoted mine in *Variant Chess* as by George, my version, on the grounds that all I had done was make a cosmetic alteration to the presentation, but George demurred, and when reprinting it in *51 Flights of Chess Fancy* I decided that "JDB after GPJ" would be a more appropriate attribution; my contribution may have been merely cosmetic, but the cosmetics of a problem are important.

This recently set me thinking about the whole question of puzzles and their cosmetics, and in particular caused me to look again at some of my favourites and to see why I find them so attractive.

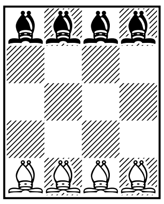
**Preliminaries**

Inevitably, the discussion that follows will disclose solutions, and although many of the puzzles will be familiar some may not be. Let me therefore start by offering an advance look at some of the examples I shall use, so that any readers who have not seen them before can have a go before the answers are thrust upon them.

We start with a couple of ancient classics. A man wishes to convey a wolf, a goat, and a basket of cabbages across a river, but the only boat available cannot carry more than one of them at a time. Given that the wolf cannot be left unattended with the goat, nor the goat with the cabbages, how does he do it?

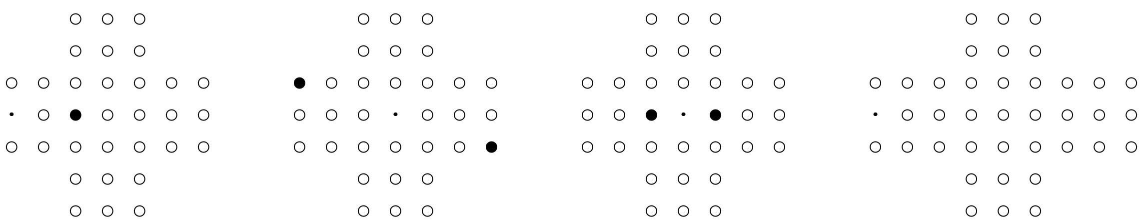
Similarly, a group of three men and their wives wish to cross a river, but the boat available can only carry two persons, and the husbands are so madly jealous that no woman can be left in the company of another man unless her husband is present. Again, how is it done?

Now to more modern material. In the position below



play to interchange the Black and White bishops without at any time allowing two bishops of opposite colour to attack each other.

If you can do the ordinary peg solitaire game, try the four problems below.



In each case the dot indicates the hole to be vacant initially, and the black pegs are to be the ones left on the board at the end (in the second and third problems, they are to change places). In the fourth problem, where the board has been extended, the final peg is to be left in the hole intially vacant, but there is no restriction on the peg which is to be left there. This may be found the most difficult of the set.

[*] In my opinion, which the dictionaries do not entirely endorse, a "puzzle" is something intended purely to entertain, whereas a "problem" may also be something whose successful solution is a matter of importance. However, the term "problem" has become so widely used for what are perhaps merely puzzles (chess problems, double-dummy bridge problems, and the like) that to try to make the distinction would be unhelpful. I have used whichever word I find more natural in the context.

At bridge, you deal yourself the hand below:

```
S  9 7 4
H  A K 9 3 2
D  A 2
C  K 6 3
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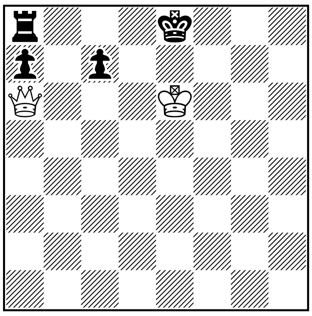
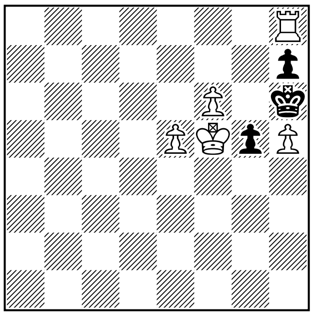
You open One Heart, West butts in with Four Diamonds, partner gives you Four Hearts, and this is passed out. West leads the queen of diamonds, and dummy goes down:

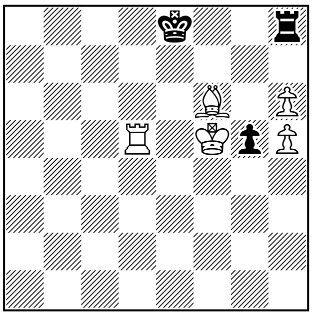
```
S  A 5 2
H  Q 10 8 6 5
D  K 3
C  A 7 4
```

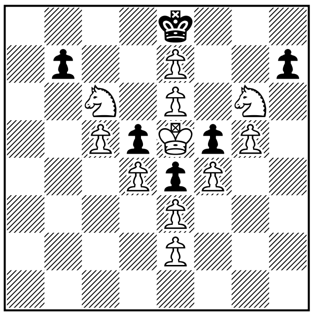
This will be laydown unless East can ruff the opening lead, but West has butted in at the Four level without either the ace or king of his suit, so the possibility that he has the rest of the suit cannot be ignored.  How do you play to give yourself a chance of still making ten tricks if this is the case?

Solve the four chess problems below.  Each is "White to play and force mate in two moves against any defence", but there is a little more to them than meets the eye.









Finally, and if you are feeling slightly masochistic, solve the equation

```
   ONE
  NINE
 FIFTY
TWENTY
------
EIGHTY
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subject to the usual rules (each letter represents a different digit, and there are no leading zeroes).

**Some problems examined**

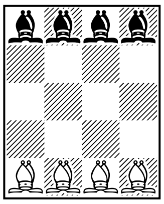
Having displayed our wares, let us examine some of them.

To solve the problem of the wolf, the goat, and the cabbages, take the goat across, come back, take the wolf across (or the cabbages, the problem is symmetrical), bring the goat back, take the cabbages across (or the wolf), and come back for the goat. It's the counter-intuitive bringing back of the goat that makes this little problem. According to David Singmaster's invaluable *Sources in Recreational Mathematics*, its first appearence was in a book *Propositiones Alcuini doctoris Caroli Magni Imperatoris ad acuendos juvenes* ("Problems to sharpen the young") written by Alcuin of York around 800.

The problem of the three jealous husbands also dates back to Alcuin. Mr A takes his wife across, and brings the boat back. Mrs B takes Mrs C across, and brings the boat back. Mr A takes Mr C across, and brings his wife back. Mr A takes Mr B across, and Mrs C brings the boat back. Mrs C takes Mrs B across, and Mr A goes back for Mrs A. This time, it is the bringing back of Mrs A that is counter-intuitive. Mrs A can be taken across at the start and finish by one of the other wives and not by her husband, but it seems only proper that the gentlemen should do most of the rowing.

There have been many developments of these (more objects or couples, islands in the middle of the river, and so on), but none seems to me to have the simple charm of the originals. "More complicated" does not necessarily mean "better".

The bishop problem

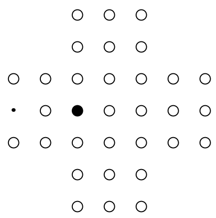
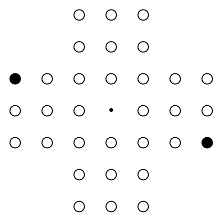
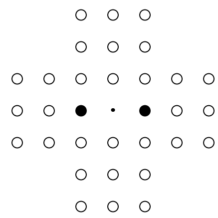
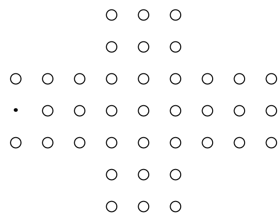


appears to consist of two independent problems, since the light-squared and dark-squared bishops never interact. If however we consider the light-squared bishops alone, we find that the problem cannot be solved unless we allow the same player to make two moves in succession. With this concession, a solution (chessboard notation, files a-d from the left, rows 1-5 from the bottom, Black's moves in bold) is c1-b2, **a5-d2**, **c5-b4**, b2-d4, **b4-a3**, **d2-c1**, d4-c3, c3-a5, **a3-c5** (the bishops at c1 and a5 have changed places, but that at c5 has gone back home and that at a1 has never moved), **c1-a3** (now we play the same moves in reverse, reflecting top to bottom, which will have the effect of interchanging a1 and c5), a1-c3, c3-d2, **c5-d4**, **a3-b2**, d2-b4, **b2-c1**, **d4-a1**, b4-c5. Dudeney, whose problem this is (number 327 in *Amusements in Mathematics*), prefaces the main problem with this simpler problem on a ten-point diamond lattice, using red and white counters instead of black and white bishops so that there is no unjustified assumption by the solver that the two colours have to move alternately.

However, we have to interchange the dark-squared bishops as well, and if we start with Black and play the diametrical reflections of the moves above (**b5-c4**, d1-a4, b1-c2, etc) we find that the light-square and dark-square solutions exactly complement each other; where the light-square solution demands a White move, the dark-square solution demands a Black, and vice versa. We can therefore alternate Black and White moves by playing the two solutions in parallel, always choosing the White move first and then the Black. This gives (dark-square moves in italics) c1-b2, ***b5-c4***, *d1-a4*, **a5-d2**, *b1-c2*, **c5-b4**, b2-d4, ***c4-a2***, *c2-d3*, **b4-a3** (everything is now down the sides), *a4-b5*, **d2-c1**, d4-c3, ***a2-b3***, c3-a5, ***b3-d1***, *d3-b1*, **a3-c5** (halfway there, the bishops at c1/a5 and d1/b5 having changed places and the others either having gone back home or never having moved), *b5-d3* (now we reverse), **c1-a3**, a1-c3, ***d5-b3***, c3-d2, ***b3-a4***, *b1-a2*, **c5-d4** (everything is down the sides again), *d3-c4*, **a3-b2**, d2-b4, ***a4-c2***, *c4-b5*, **b2-c1**, *a2-d5*, **d4-a1**, b4-c5, ***c2-b1***.

One of Dudeney's best, in my opinion.

Peg solitaire problems on a square lattice use an algebraic notation similar to that which is used for chess, but for solitaire we put row 1 at the top. However, when the initial vacancy is on row 4 and the survivors finish on the same row, it doesn't matter from which end we count; the resulting solution, although reflected top to bottom, will still be valid.

Vacate a4, put a black peg at c4, and play to leave this as the last peg. The gimmick here is that the black peg must finish far away at g4 (this can be proved analytically, but here is not the place to do so). The simplest solution is probably **c4-a4**; c2-c4, a3-c3, c4-c2; c1-c3, d3-b3, e2-c2, e1-c1-c3, b3-d3; e4-e2, g3-e3, d3-f3, g5-g3-e3, e2-e4; c6-c4, d4-b4; a5-c5; d6-d4, f5-d5, d4-d6; e7-e5, e4-e6, c7-e7-e5; **a4-c4-c6-e6-e4-g4**. This is from an anonymous book *Der praktische Solitärspieler*, published in München in 1808 and drawn to my attention by Dic Sonneveld (I have reordered the jumps). A solution finishing straight across the centre (**a4-c4-e4-g4**) is also possible.

Vacate d4, put black pegs at a3 and g5, and play to interchange them and clear the rest of the board. b4-d4; c6-c4, a5-c5, c4-c6, c7-c5, d5-b5; d3-d5, f4-d4, d5-d3; f5-d5, e7-e5, d5-f5; d7-d5; e2-e4, g3-e3, e4-e2, e1-e3, d3-f3; b3-d3, c1-c3, d3-b3; d1-d3; **a3-a5**, **g5-g3-e3-c3-a3**, **a5-c5-e5-g5**.

Vacate d4, put black pegs at c4 and e4, and play to interchange them similarly. d2-d4, b3-d3, c1-c3, **c4-c2**; d4-d2, d1-d3, e3-c3; e1-e3, **e4-e2**, g3-e3; e6-e4, g5-e5, e4-e6; e7-e5, g4-e4-e6, c7-e7-e5; c6-c4, b4-d4; a5-c5, d5-b5, a3-a5-c5; **c2-c4-c6**, **e2-e4-c4**, **c6-e6-e4**.

On the extended board, vacate a4 and play to leave the last survivor there. c4-a4, e4-c4, e2-e4, c3-e3, a3-c3, f3-d3-b3; f1-f3, f4-f2, e5-e3, d1-f1-f3-d3; a5-a3-c3-e3; h3-f3-d3, d2-d4-b4; d6-d4; g5-e5, i5-g5, b5-d5-f5-h5; f7-f5, h4-f4-f6, d7-f7-f5; i3-i5-g5-e5; e6-e4-c4-a4. Whereas the solutions to previous problems have allowed a measure of variation, this solution is unique to within symmetry and jump order; if you separate the moves of your solution into individual jumps, do the same with the solution here, and tick off the jumps one by one against each other, you will find that you have made either exactly the same jumps or a top-to-bottom reflection of them.

To give yourself a chance of making ten tricks on

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                S  A 5 2
                H  Q 10 8 6 5
                D  K 3
                C  A 7 4
D Q led
                S  9 7 4
                H  A K 9 3 2
                D  A 2
                C  K 6 3
```

even if East ruffs the opening lead, cover the queen of diamonds with the king, and if East ruffs throw your ace as well. You can now still succeed if West has exactly one spade and either one or two clubs. If this is the case, you can win East's return, draw trumps, cash your black-suit winners, and exit with the two and three of diamonds. West will have to win, and having nothing else left will have to lead another diamond. Throw a club from dummy and a spade from hand, leaving West on play. West will have to lead a fourth diamond; ruff in dummy and throw a second spade from hand, and make the rest on a cross-ruff. Despite having started with exactly the same distribution as your partner, you have made three tricks by cross-ruffing.

I saw this many years ago in (I think) *Bridge is Only a Game* by Hubert Phillips. I am quoting from memory, but I think the essentials are correct.

There are several reasons why I find these problems attractive.

Alcuin's problems have amusing settings and piquant twists in their solutions (the counter-intuitive bringing back of the goat and of Mrs A). They are so familiar that their difficulty is hard to assess, but I think most people find them tricky on a first acquaintance, and are pleased when they either work out or are shown the solutions.

Dudeney's bishop problem has a neat and natural setting, is unexpectedly difficult, and has the pleasant feature that the second half of the solution is an exact reversal of the first. Dudeney appears to have thought it too difficult to be set in isolation, and prefaced it with a simpler problem to set the solver on his way. I suspect that many readers will agree.

The peg solitaire problems offer various attractive features. The appeal of the first surely rests in the fact that the black peg must finish so far away. That of the second rests in its extreme task of interchanging two men at opposite corners of the board. The third problem features a curious little task and a perhaps surprising level of difficulty, while the fourth is here primarily on account of its difficulty (it is the only single-vacancy single-survivor problem I know, on a board of natural shape and reasonable size, whose solution is unique to within symmetry and jump order). If you got this fourth problem out in under two hours, you did well.

The bridge problem is a gem. On the face of it, we have ten winners (five trumps, ace and king of diamonds, ace and king of clubs, ace of spades) and three inescapable losers (two spades and a club). The ruff kills one of our diamond winners and we still have our spade and club losers to come, but we throw away our other diamond winner and give the defence not just one but two further tricks in diamonds, and our spade and club losers go away on them. I cannot say how difficult it is because I saw it in a book and had no chance to solve it as a puzzle, but it has stuck in my mind for the best part of forty years.

In summary, I find these problems attractive because they embody some or all of the following: a tempting setting, an elegant or piquant solution, and an enjoyable level of difficulty.

**Discovering and inventing new puzzles**

On the face of it, all that is needed to discover or invent a new puzzle is to look for something which appears to be novel and incorporates as many as possible of the features that we have just highlighted. However, "an enjoyable level of difficulty" is a property of the solver as much as of the puzzle; what one solver finds enjoyably difficult, another may find almost trivial, while a third will not even know how to begin. You have absolutely no control over this, and all you can hope for is to discover something which will be enjoyed by all, even by those who give up and look at the answer.

One way of finding good puzzles, particularly puzzles based on games, is to search systematically through problems of a particular type and see if any seems particularly interesting. The author of *Der praktische Solitärspieler* made an extensive study of problems of the form "vacate *X* and mark the man at *Y* to be the final survivor", and I merely selected the first peg solitaire problem here as being the most piquant of them. I have myself made a study of "swap" problems where the hole initially vacated is midway between two men who have to be interchanged, and I found, as one typically does when doing a search of this kind, that some were very easy, some were clearly unsolvable, some were enjoyably difficult, and one or two were unsolvable but very hard to prove so. For present purposes, I picked two that I found enjoyably difficult. It is only theoreticians who are interested in proofs of unsolvability, however subtle; the ordinary puzzle enthusiast wants something he or she can do.

Another way of producing good puzzles, or at least amusing puzzles, is to take an existing idea and reclothe it. The technique is particularly appropriate to logical problems: the sort of puzzle where a town contains a butcher, a baker, and a shopkeeper who sells electric light fittings (we don't have candlestick makers any longer), they are named Smith, Jones, and Robinson in some order, one plays bridge, one poker, and one cribbage, one is married to a blonde, one to a brunette, and one to a redhead, and so on, and we are given various pieces of information and have to work out which is which. The point is that a statement that Robinson is not the baker is logically equivalent to a statement that Mrs Robinson is always making snide remarks about the baker's wife's hats, so the puzzle can be restated in this more exotic form without changing anything. The scope for this sort of thing is almost limitless.

It is also possible to take and expand an existing idea. I remarked earlier that making a puzzle more complicated did not necessarily make it better, but this is no reason for not experimenting. In the classic "twelve coins" problem, you are given twelve coins and a balance, you are told that one of the coins is a dud and is either too heavy or too light, and you have three weighings to find out which coin is wrong and whether it is heavy or light. Many years ago, it occurred to me to see if this could be expanded to take account of two duds. To simplify matters, I stipulated that two heavy duds should balance each other, that two light duds should also balance, and that one heavy and one light dud should balance two normal coins. If we also allow there to be only one dud or to be no dud at all, this gives $2n^2 + 1$ possible cases for $n$ coins, and for $n = 11$ this is 243 which is $3^5$.

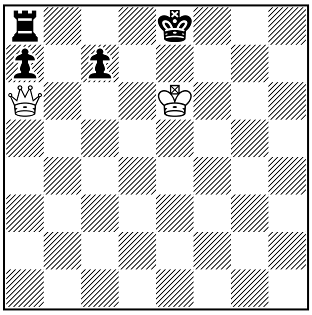
So can we identify the dud or duds among eleven coins in five weighings? I found that if I weighed 1 2 3 against 4 5 6, each of the three possible results could arise in exactly 81 ways. I then found that if I added weighings of 1 3 4 against 2 7 8 and 1 4 2 against 3 9 10 (in other words, if I cycled 2 3 4 round and put two fresh coins on the right-hand side each time), each of the 27 possible results could arise in exactly nine ways. The natural next weighing to add was of 5 7 9 against 6 8 10, and lo and behold, each of the 81 possible results could arise in exactly three ways. A fifth weighing to distinguish between these three possibilities was easily set up in each case, so to my delight the problem could be solved.

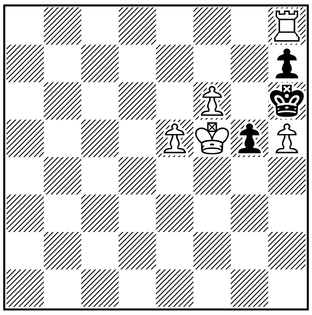
The conditions of this problem are somewhat artificial, and I do not think it can in any sense be regarded as "better" than the classic twelve-coins problem. It has however long been among my favourites, even though I cannot claim any particular credit for its discovery. I just looked, and there it was.

And even ordinary mathematical problems can be set as puzzles, particularly when the intended audience is young. When I was ten or eleven, our primary school teacher gave us a puzzle along the following lines: if two apples and three pears cost one and a penny (pre-decimal coinage, twelve pennies equalled one shilling) and five apples and two pears cost one and fourpence, how much will three apples and four pears cost? It wasn't until a couple of years later that I realised that I had been introduced to simultaneous linear equations through the medium of a puzzle long before I had met ordinary linear equations in formal algebra lessons.

**Tricks and teases**

In the problems so far, everything the solver needs to know is explicitly stated. In the problems that follow, something not explicitly stated has to be deduced.

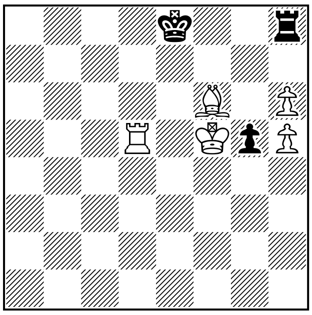


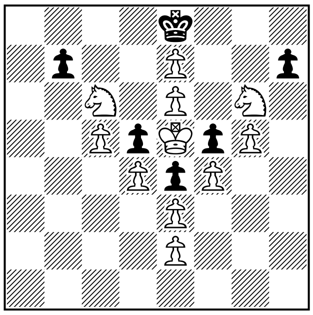


In the first chess problem (by Sam Loyd, *Musical World* 1859), 1 Qa1 will lead to mate by 2 Qh8 unless Black castles. But in the given position it is White's move, so Black must just have moved, and since neither of his pawns has moved this move must have been made by either the king or the rook. So Black has lost the right to castle, and Qa1 does indeed force mate next move.

The second problem (Friedrich Amelung, *Düna Zeitung* 1897, refining an idea apparently first shown by Max Lange) looks quite impossible, but again Black has just moved; what was this move? Not by the king from g7, since the pawn on f6 would have been giving check and there is nowhere whence this pawn could have come. Obviously not by the king from g6, and not by the pawn from g6 because it would have been giving check to the White king with Black to move. So it must have been by the pawn from g7, and White can take it *en passant* (1...Kh5 2 RxP).†

† Lange's problem (in Forsyth notation, N7/1p1Bpp2/1P1k4/1KpP4/4R1N1/B7/8/8, mate in three) appeared anonymously on the front cover of *Schachzeitung* in February 1858, and its attribution to Lange is due to Hans-Peter Suwe (*Quarterly for Chess History* 10, page 277). I had suggested that it was the work either of Lange or of Adolf Anderssen, Lange being the editor of *Schachzeitung*, Anderssen being one of his contributors, and both being problemists, but Hans-Peter argued convincingly that it was in fact the work of Lange, and I now associate myself with his opinion. *Düna Zeitung* was a German-language newspaper or periodical that circulated in and around Riga, "Düna" being the German name for the river through the city.





The third chess problem (W. Langstaff, *Chess Amateur* 1922) combines these ideas. If Black's last move was with his king or rook, he cannot castle, and 1 Ke6 forces mate next move (2 Rd8). If it was with the pawn, this pawn must have come from g7, and 1 PxP *en passant* forces mate (the threat is again 2 Rd8, and if 1...0-0 then 2 h7). White can always force mate in two, so the problem is fairly posed, but in the absence of further information we cannot say which solution is needed.

And is not the fourth problem (T. R. Dawson, *Falkirk Herald* 1914) more of the same, White capturing *en passant* on d6 or f6 according to Black's last move? No! White must have made ten captures by pawns (in some order, axb, bxc twice, cxd twice, dxe twice, fxe, gxf, hxg), and Black's ten missing men, which must have provided the fodder for these captures, include his light-square bishop. So d7 must have been vacated long ago to let this bishop out, and Black's last move must unambiguously have been f7-f5.

An important point concerning these problems is that the required condition (that Black has moved his king or rook or that his last move was a pawn-two) can be *proved* (or, in the case of the third problem, that one or the other must be true even though we have no way of telling which). It is *not* enough simply to present a position with (say) a White pawn on d5 and a Black pawn on e5, and say "ah, let us assume that Black has just played pawn-two, then White can capture *en passant* and this gives us a solution to the problem". The pawn-two move to validate an *en passant* capture, or the existence of a previous move by king or rook if we are relying on Black's inability to castle, must be *provable*.

Animals-in-field problems provide another example where something not explicitly stated may have to be deduced. These can be thought of as variants of cistern problems, the sort of problem where we are given an empty cistern with three taps, we are told how long the cistern takes to fill if we turn on taps A and B, or if we turn on taps B and C, or if we turn on taps C and A, and we have to work out how long it takes if we turn on all three (we have three equations in three unknowns). In an animals-in-field problem, we have three animals eating a field bare and at first sight the problem is the same, but in practice the grass will grow while the animals are eating it; what should we do about this? Dudeney, in problem 231 of Martin Gardner's 1967 edition *536 Puzzles & Curious Problems*, tells the solver to assume no growth. Loyd, in his 1914 *Cyclopedia of 5,000 Puzzles*, pages 47 and 345, adds a fourth equation from which the rate of growth can be calculated. But should we explicitly warn the solver to allow for the grass's growth, or can we simply present him with four equations and the word "grass", and leave him to realise that he must allow for growth if the equations are not to be inconsistent? Loyd thought the appearance of the word "grass" was sufficient and I agree, but I suspect that some readers may take a different view.

In the last resort, it all boils down to this: if the solver of a trick problem fails and has to look up the answer, will he be amused when he sees it, or will he feel cheated?

**The role of the computer**

In my student days, I was shown the problem

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   ONE
  NINE
 FIFTY
TWENTY
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EIGHTY
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to be solved according to the usual rules (each letter represents a different digit, and there are no leading zeroes). According to David Singmaster's *Sources*, it was invented by Alan Wayne, and first appeared around 1945 in *The Cryptogram*, an American puzzle magazine. I eventually got it out (I haven't seen the author's solution, and outline an improved version of my own in the Appendix), and it occurred to me to try the "Go forth and multiply" problem

```
   HOGS
   SOWS
-------
PIGLETS
```

subject to the same conditions (it too uses all ten digits). An hour with a calculating machine gave me

```
   2591
   1581
-------
4096371
```

as a unique solution, and I thought what a good puzzle it would make. I would now say emphatically that it does *not* make a good puzzle; a good puzzle should not require the solver to resort to a computer. The addition sum ONE + NINE + FIFTY + TWENTY = EIGHTY makes a good puzzle, if maybe at the limit of acceptable difficulty. HOGS x SOWS = PIGLETS is merely a curiosity.

But if the computer should have no role in the *solving* of puzzles, it certainly has a role to play in their *discovery*. When trying out peg solitaire on a new network, I always start by asking a computer which of the single-vacancy single-survivor problems can be solved. I don't ask it for the solutions; if I know that a problem can be solved, I am happy to sit down and work out the answer. But proving that a problem is unsolvable can be quite difficult, and the preliminary computer check saves me from wasting time and effort on non-starters.

Again, a computer can help by grinding through all the positions of a certain type and highlighting the potentially interesting ones. We have already seen that the first three peg solitaire problems above were discovered by systematic search (the first by a study by the author of *Der praktische Solitärspieler* of all problems of the kind "vacate *X*, mark the peg at *Y*, and play to leave this as the final survivor", the next two by myself of all problems of the kind "Vacate *X*, mark two pegs at equal and opposite distances from *X*, and play to interchange them"), but these searches were done by hand. The fourth problem was also found by systematic search, but this time the discovery was by computer. George Bell asked his computer to make a systematic investigation of single-vacancy single-survivor problems on boards comprising a 3x3 square with a 3x*n* arm on each side, these arms not necessarily being of the same length, and he found that this one took an inordinate time to resolve. The subsequent proof that the solution was unique to within symmetry and ordering of jumps was done by hand, but it was George's computer, by taking abnormally long to solve the problem, which had indicated that it might be worth investigating.

**Anticipation and plagiarism**

These are important matters, and let us make an important distinction. *Anticipation* occurs when somebody publishes a puzzle in good faith, honestly believing himself to have been its discoverer, only to find that someone else had got there first. *Plagiarism* is the deliberate copying of somebody else's work and passing it off as one's own.

Being anticipated is no crime. It already happens quite frequently, and will happen ever more frequently as more and more is discovered and published. I rediscovered the first peg solitaire problem during our researches at Cambridge in 1961-62, and put it in *The Ins and Outs of Peg Solitaire* without acknowledgement in the belief that it was new (I didn't become aware of *Der praktische Solitärspieler* until a few years ago). Many similar examples could be adduced.

But even though anticipation becomes more of a problem every day, it is surprising how many things that "must surely have been discovered before" appear not to have been. The problem of a knight's tour on a chessboard dates back to the ninth century, and since the 1840s people have been looking for "magic" tours in which the squares are numbered from 1 to 64 and the numbers in each row and column add to 260. The ultimate task, in which the diagonals also add to 260, proved elusive (and has now been demonstrated by computer search to be impossible), but several tours have the property that the two diagonals together add to 520. One such is an elegant tour which was first published by C. F. Jaenisch in *Chess Monthly* in 1859 and has since been widely quoted (ignore the diagonal arrows for the moment):

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 256 | | 256 | | 256 | | 256 | | | | |
| | ↘ | | ↘ | | ↘ | | ↘ | | | |
| | | 27 | 30 | 51 | 40 | 53 | 2 | 15 | 42 | ← 260 |
| | | 50 | 39 | 28 | 31 | 14 | 41 | 54 | 3 | ← 260 |
| | ↗ | 29 | 26 | 37 | 52 | **1** | 56 | 43 | 16 | ← 260 |
| 264 | | 38 | 49 | **32** | 25 | 44 | 13 | 4 | 55 | ← 260 |
| | ↗ | 23 | 36 | 45 | 12 | 57 | **64** | 17 | 6 | ← 260 |
| 264 | | 48 | 11 | 24 | **33** | 20 | 5 | 58 | 61 | ← 260 |
| | ↗ | 35 | 22 | 9 | 46 | 63 | 60 | 7 | 18 | ← 260 |
| 264 | | 10 | 47 | 34 | 21 | 8 | 19 | 62 | 59 | ← 260 |
| | ↗ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | |
| 264 | | 260 | | 260 | | 260 | | 260 | | |
| | | | 260 | | 260 | | 260 | | 260 | |

This tour is re-entrant (it can be carried forward from 64 to 1), rotationally symmetric (the numbers from 33 to 64 are diametrically opposite to those from 1 to 32, so when the pattern is drawn out and rotated through 180 degrees it remains the same), and magic (each row and column adds to 260), and when quoting it in *Variant Chess* in 2008 I remarked that although the diagonals did not add to 260, the principal odd diagonal *and each parallel odd broken diagonal* added to 256, and the principal even diagonal *and each parallel even broken diagonal* to 264. I said that I could not believe that this had not been noticed before, but that I was not aware of any evidence to the contrary (I had been through all the obvious sources, including an unpublished monograph by H. J. R. Murray held in the Bodleian Library). Nobody wrote in to draw attention to a previous mention, nor did anybody when I repeated the question to a wider audience in the January 2012 issue of *The College Mathematical Journal* (a tribute issue to Martin Gardner). So it is just possible that this additional property, which to my mind significantly increases the elegance of the tour, passed unnoticed until I drew attention to it in 2008.

Plagiarism is a totally different matter, and can be passed over very briefly. Editors are not fools, they have access to substantial libraries of reference material, and a plagiarist is normally soon spotted. Using an existing puzzle as a basis is another matter and is in my view legitimate provided that your contribution is significant, but it is proper to acknowledge any source of which you have knowingly made use.

**Summary and conclusions**

This little essay was prompted by a remark that the cosmetics of a chess problem were important, but it seems to have become a general discussion of puzzles and their aesthetics. In particular:

- the setting of a puzzle should be such as to tempt the solver to have a go;
- a puzzle should be enjoyably difficult, but should not require the solver to use a computer;
- the solution should contain something elegant or piquant to reward the solver for his efforts;
- even a solver who gives up (for what is “enjoyably difficult” to one solver may be far beyond the capability of another) should derive pleasure from seeing the answer;
- if a puzzle depends on something which is not explicitly stated but has to be inferred, a solver who fails and looks up the answer should feel amused rather than cheated;
- being honestly anticipated is no crime, but you should acknowledge any source of which you have knowingly made use.

And if you are tempted to try thinking up puzzles of your own, good luck. When I was a chess columnist, I was sometimes asked by a budding composer how many problems or endgame studies he needed to produce for his efforts to have been worthwhile, and my answer was always the same: “One good one.” I think the same is true here.

**Appendix**

```
   ONE
  NINE
 FIFTY
TWENTY
------
EIGHTY
```

We start by looking at the carries from the hundreds column westwards. The carry from the hundreds to the thousands cannot exceed 3. $N + I + E$ cannot exceed 24, since $N$, $I$, $E$ are all different, so the thousands cannot add to more than 27 and the carry out of the column cannot exceed 2. $F + W$ cannot exceed 17 and the carry in cannot exceed 2, so the tens-of-thousands cannot add to more than 19 and the carry out cannot exceed 1. So $E = T + 1$.

Now let us look at $NE$ and $TY$. $NE + NE + TY$ must equal 100 or 200, and we have just seen that $E$ must equal $T + 1$. If we run through all the values of $NE$ from 00 to 99, we find that the only possibilities are 42 + 42 + 16, 59 + 59 + 82, 84 + 84 + 32, and 92 + 92 + 16. These give the four cases

```
   O42       O59       O84       O92
  4I42      5I59      8I84      9I92
 FIF16     FIF82     FIF32     FIF16
1W2416    8W9582    3W4832    1W2916
------    ------    ------    ------
2IGH16    9IGH82    4IGH32    2IGH16
```

where “O” is letter O and not figure 0.

Next, we observe that $ONE$ gives the same remainder as $O + N + E$ on division by 9, and similarly for the other numbers. It follows that $4N + 3E + 3T + 2F + 2I + 2Y + O + W$ gives the same remainder as $E + I + G + H + T + Y$, or, adding $F + N + O + W$ to each side, that $5N + 3E + 3T + 3F + 2I + 2Y + 2O + 2W$ gives the same remainder as $E + I + G + H + T + Y + F + N + O + W$. But this last expresson includes all ten digits, each once and once only, so it adds to 45 which gives remainder 0, and so $5N + 3E + 3T + 3F + 2I + 2Y + 2O + 2W$ must also give remainder 0. Multiplying by 4, noting that $8X$ is equivalent to $-X$, and rearranging, we see that $O$ must be the remainder when $(2N + 3E + 3T - Y) + 3F - I - W$ is divided by 9, a remainder of 0 causing $O$ to be 9 since it cannot be zero.

For each of the four cases above, we now note down $N$, $E$, $T$, $Y$, the remainder when $2N + 3E + 3T - Y$ is divided by 9, and the six digits not yet used. We then take each of these six possible values of $I$ in turn and each possible carry out of the thousands column (this carry may in principle be 0, 1, or 2, but at most two values will be possible for any particular value of $I$), compute $F + W$ (which will be $I + 10$, $I + 9$, or $I + 8$ according as the carry from the thousands column is 0, 1, or 2), look in our list of available digits for suitable values of $F$ and $W$, and use the formula $(2N + 3E + 3T - Y) + 3F - I - W$ to see what $O$ must then be. If $O$ is a digit still available, we use the hundreds column to calculate $H$, and if this too is a digit still available we use the thousands column to calculate $G$, and we verify that the carry out of this column is what we had assumed it to be.

For example, consider case 1 ($N = 4$, $E = 2$, $T = 1$, $Y = 6$). $2N + 3E + 3T - Y$ gives remainder 2 on division by 9, and we have digits 0/3/5/7/8/9 still available. Start with $I = 0$ and assumed carry 0 out of the thousands column. $F + W$ must be 10, so $F$ and $W$ must be 3 and 7 in some order. Try $F = 3$, $W = 7$: no, the formula says that $O$ must be 4, which is a value we have already used. Try $F = 7$, $W = 3$: no, $O$ must be 2, still no good. Try carry 1 out of the thousands column: no, not possible with $I = 0$. All right, try $I = 3$ and assumed carry 0. We now have $F + W = 13$, so $F$ and $W$ must be 5 and 8. Try $F = 5$ and $W = 8$: no, $O$ must be 6, again no good. Try $F = 8$ and $W = 5$: now $O$ must be 9, which is possible, but $H$ turns out to be 5 which clashes with $W$, so yet again no good.

Proceeding systematically in this manner, we eventually come across in case 3 ($N = 8$, $E = 4$, $T = 3$, $Y = 2$)

```
   984
  8584
 75732
364832
------
450132
```

and if we continue to the end we find that this solution is unique.