

Some 18th Century Books on Mathematical Recreations

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(A paper written for a meeting on the history of recreational mathematics held at South Bank University, London, on 24 October 1992, and slightly revised following reactions at the meeting and subsequent comments)

When David Singmaster invited me to talk to this meeting, he suggested that I might choose a topic within the history of board games, but I have been concentrating on other matters since 1984 and could say virtually nothing that has not already appeared elsewhere. As an alternative, I suggested that a brief description of some of the books which I encountered in my 1984 investigations might be of interest, if only as a pleasantly undemanding talk to fill the half-hour immediately after lunch. I don't claim to have seen every book that was published, let alone to have collated all the different editions; that would be a task for a different researcher, to be reported on a different occasion. But I think the descriptions that follow will give a fair picture of what was available in the late eighteenth century to the person who now reads Martin Gardner.

I shall describe three books: Ozanam, *Récréations mathématiques et physiques* (4 volumes, 1850 pages); Guyot, *Récréations physiques et mathématiques* (3 volumes, 1130 pages); and Wiegleb, *Unterricht in der natürlichen Magie* (19 volumes, 7050 pages). These add up to over 10,000 pages and I have not included plates and indexes, so my portrayal will be made with a very broad brush. The books borrow extensively from each other and use common sources, and they also have this in common: each appears to have been popular in its day. Today, the picture is a little different. Ozanam remains well known, if perhaps more by repute than by personal acquaintance; Guyot is little known; Wiegleb is hardly known at all. (Enquiry at the meeting disclosed five people among an audience of approximately one hundred who actually owned an edition of Ozanam, plus another six who had at some time read from an edition in a library; one person who owned an edition of Guyot, plus one who had read from an edition; and nobody at all who had set eyes on Wiegleb.)

Let us therefore start with Ozanam. It went through numerous editions from 1694 onwards, some in translation, but I shall concentrate on the edition of 1750. In this edition, volume 1 contains approximately 270 pages on arithmetic and similar topics, 60 on geometry and mensuration, a dozen on music, and 115 on optics; volume 2, 130 pages on sundials and gnomonics, 205 on astronomy and related topics, and 130 on mechanics; volume 3, 100 pages on fireworks, 200 on physics, and 180 on clocks and horology; volume 4, 210 pages on natural phosphors, 60 on artificial phosphors, 130 on perpetual lamps, and 55 on conjuring tricks and puzzles. All of this will have made fascinating reading for anyone with an enquiring mind, but our primary concern here is with the chapter on "arithmetic" in volume 1. It starts with three classic puzzles:

- the blind abbess and her illicitly visited nuns;
- the wolf, the goat, and the cabbage;
- the three jealous husbands.

There follow various expositions (of amicable numbers, prime numbers, triangular numbers, arithmetical and geometrical progressions, magic squares, Pascal's triangle, permutations and combinations, and the probabilities of various dice throws), and then some more puzzles:

- some women sell vegetables at various prices, but they take the same amount of money and it is required to find who sells how much at each price;
- pipes fill cisterns at different rates, and the task is to calculate what happens;
- numbers are required which yield certain remainders;
- a person is asked to think of a number, and then to perform an apparently bewildering calculation which in fact discloses it;
- 8 pints of wine are to be divided equally using glasses holding 5 and 3 pints;
- the contents of 7 full, 7 empty, and 7 half-empty barrels are to be divided equally between three people;
- a knight is to make a complete tour of a chessboard.

There is much more, as befits a chapter of 270 pages, but these extracts give the flavour.

Also of interest for present purposes is the final chapter of volume 4. Part of this chapter deals with sleight of hand, but there are also some first-class topological puzzles. Examples:

- threading paired cherry stalks through impossibly small holes without removing the cherries;
- extracting a pair of scissors from a loop of string;
- untangling two people linked by ropes knotted over their wrists;
- extracting rings from looped cords (the puzzle known as “Solomon’s Seal”);
- disentangling wires encumbered with rings (the “Chinese Rings”).

For several of these puzzles, earlier editions of Ozanam appear to be the oldest known sources. It may be added that the chapters on mechanics and physics also contain tricks of various kinds.

So much for Ozanam 1750. The 1770 edition appears to be essentially the same, but the 1778 edition shows extensive revision. Volume 1 now contains arithmetic and geometry; volume 2, mechanics, optics, acoustics; volume 3, astronomy, geography, gnomonics, navigation, architecture, fireworks; volume 4, physics, magnetism, electricity, chemistry. The chapter on arithmetic is much more methodical, and a section on “political arithmetic” (statistics) has been added. My notes say that the knight’s tours are omitted from this edition and I said as much during the lecture, but David Singmaster pointed out that they are in his 1790 edition and a subsequent comparison of this edition’s page numbers with my notes disclosed no obvious discrepancy. I therefore suspect an oversight, for which I apologize. What is certainly true is that the topological puzzles are absent from the 1778 edition, which is a pity.

Guyot’s book, which was first published in 1769, also went through several editions, two being in German and Italian. It is much less readily available than Ozanam, but copies of the 1786 and 1800 editions are in the Bodleian Library. Ozanam calls his book “Mathematical and physical recreations”; Guyot’s recreations are “physical and mathematical”, and the change in emphasis is noticeable. Volume 1 contains 46 recreations on magnetism and 61 on electricity; volume 2, 10 on geometry, 54 on light, 15 using fire, 18 on pneumatics, 8 using hydrogen, and 8 on hydraulics; volume 3, 53 on numbers, 68 on sleight of hand, 18 on mechanics, 15 on hidden writing, and 4 on elementary cryptography. There is a strong emphasis on tricks and trick effects (particularly in the chapter on magnetism) and some of the recreations using fire and hydrogen strike me as distinctly dangerous. The chapter on numbers, apart from routine matters, includes “think of a number” problems, the 8/5/3 wine problem, Pascal’s triangle, chess problems (one and possibly all of which are taken from Salvio’s *Trattato dell’inventione et arte liberale del gioco degli scacchi* of 1604), knight’s tours, and magic squares. On the whole, however, this chapter is not of great interest; Guyot is much better on science than on mathematics. It may be added that the drawings are superb.

Now to Wiegleb. In 1751, the Berlin publisher Nicolai produced a book by one Martius, *Unterricht von der Magia naturalia*, which appears to have consisted of magical and mythical material of various kinds. No doubt it contained little or nothing of value, but it sold out, and a reprint was demanded. In 1779, therefore, Nicolai published Wiegleb’s *Unterricht in der natürlichen Magie*, which claimed on its title page to be a “fully revised” edition of Martius but in fact was a completely new book in the tradition of Ozanam and Guyot. This 1779 edition was a single volume of 416 pages, but a second edition in 1782 was expanded to 461 pages. There followed, in 1783, a book *Natürliche Magie* by Funk, Professor of Natural Philosophy at Leipzig, of which I know no copy in this country though there are two in America (in the Library of Congress and in the Joseph Regenstein Library of the University of Chicago); in 1786, a second volume of Wiegleb; from 1789 to 1805, a third edition of volume 1, a second edition of volume 2, new volumes 3 to 19 inclusive (volume 6 onwards being written by one Rosenthal though Wiegleb’s name was retained), and an index to the whole; and in 1806, the supply of Wiegleb-Rosenthal having ceased, a second edition of Funk. At this point, Napoleon defeated the Prussians at Jena, which is uncomfortably close to Berlin, and I do not know what happened afterwards.

Unlike the volumes of Guyot and Ozanam, each of which has its own topics, a volume of Wiegleb contains eight to ten chapters whose topics are drawn from a range of standard subjects: electricity, magnetism, optics, chemistry, mechanics, mathematics, agriculture, botany, communication and cryptography, card tricks, and games. The chapters on card tricks and games are usually short, and that on games is always last. The mathematical chapters average thirty pages per volume, and cover a lot of ground: geometrical constructions, problems whose solutions demand algebraic reasoning, numerical phenomena (primes, progressions, permutations, magic squares, difference tables), arithmetic to various bases (2, 4, 12), algorithms, logarithms, construction of calculating machines, finger semaphores for digits, puzzles, and curiosities. This is a much wider range than is offered by either of the other books, though the distribution of the material throughout so many volumes inevitably leads to bittiness. The treatment of games is likewise fragmented and no particular game appears to be covered more than superficially, but among the games considered are chess (knight’s tours,

and a few studies similar and perhaps identical to those I remembered from Guyot), other board games (solitaire, fox and geese, and a “kriegspiel” played on an 11x11 board), card games (whist, piquet, and several others), ball games (billiards, rackets, skittles), dice games, dominoes, and miscellanea.

In so far as I have investigated Wiegler, it appears to be characterized by breadth rather than by depth, but its author cites Guyot (whose science is more interesting than his mathematics) as the best of his sources, and it is possible that the chapters over which I have passed are much better than those at which I have looked. It is available in the British Library, though not always with ease. (The most convenient set is that which is catalogued as “1395.b./”, but as of October 1992 ten of the nineteen volumes have lost the binding over their spines and the spine of an eleventh is flapping ominously. In the absence of the spine, the only way to identify the book is to take it out and look inside, and at times of stress the staff may simply deliver the volumes which still have spines and report that this is all that is on the shelf. It is then necessary to explain at the Book Delivery enquiry desk that the whole set does exist and has been successfully delivered in the past.) The index, which is nominally “volume 20”, is bound with volume 19. [The above was written nearly twenty years ago, and the set may since have been rebound; I haven’t been back to check.]

There remains one matter which may be of interest: how much did these books cost? In the case of Guyot 1786, we know, because the price is included in a bookseller’s foreword. The three volumes, “brochés”, cost 21 livres; “reliés”, 24 livres; and with figures “enluminées”, an extra 6 livres. Modern dictionaries are not always helpful in respect of ancient technical terms, but I suspect that “brochés” meant with a basic paperback binding and “reliés” with a proper leather binding, and that figures “enluminées” were hand-coloured. Additionally, much of the equipment described in the book could be bought. A typical price was 5-10 livres; the highest, a remarkable 240 livres.

A sum of money is meaningless in itself, of course; we need to know how long it took to earn, and what else it would buy. In Paris, in 1789, an ordinary workman in employment could expect to earn 15-20 sous per day, and a carpenter 2 livres; 20 sous made up 1 livre. In the provinces, in 1786, a loaf of bread cost less than 3 sous, and a bottle of wine around 4 sous. These figures must be treated with some caution, because there was a price inflation of around 60% between 1786 and 1789 and everything cost more in Paris; a factory owner who claimed that a workman’s family could live perfectly well on 15 sous a day provoked a distinctly hostile reaction. Even so, they make reasonable sense. No doubt most of the purchasers came from the leisured and professional classes, but suppose that an intelligent and literate young carpenter or similarly paid artisan could set aside a tenth of his income; four or five months would suffice to buy a set of Guyot. The cost of a piece of equipment will have reflected labour, materials, and the usual middlemen’s profits; no doubt instrument makers earned more than ordinary carpenters, so no direct deduction is possible, but the prices seem about right judging from the drawings. Even the price of Guyot itself appears reasonable, allowing for the cost of hand-laid paper and hand-set letterpress; and the editions sold out, so the market must have been there.

So: three wide-ranging books, of which this half hour has given only the broadest of pictures. I hope that some of my hearers may feel inclined to investigate them personally.