The Chess Endgame Studies of Richard Réti : Knights and pawns
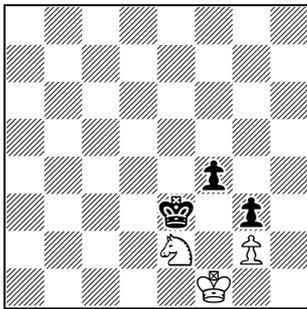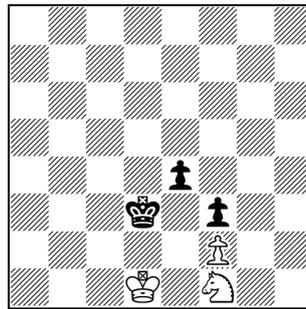
John Beasley, 14 January 2012

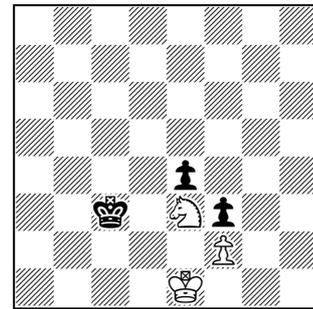| 2.1 (M 5, with Artur Mandler)	2.1a (Trinks)	2.1b
White to play and win	White to play and win	After 2...Kc3

2.1 and **2.2**, composed with Artur Mandler, appeared as a twin study in *L'Eclaireur de Nice* in 1924. In fact they are difficult alternative solutions to a study by O. Trinks which had appeared shortly before in *Oesterreichische Schachrundschau*. By modifying Trinks's position slightly and shifting it one file each way in turn, Mandler and Réti created two much deeper studies each with a unique solution.

Let us therefore first look at Trinks's study, which is **2.1a**. The strategic objectives are simple enough. White wants to advance his king to d2, after which the win will be easy; Black wants to play ...e3 and exchange off White's last pawn. Trinks therefore played 1 Ne3 Kc3 2 Ke1 (see **2.1b**) Kd3 3 Nd1 Kc2 4 Nb2 (a startling but safe move, since 4...Kxb2 will allow 5 Kd2 and White will mop up) Kc1 (4...Kc3 5 Kd1 with Kd2 to follow) 5 Na4 Kc2 6 Nc5 and the e-pawn goes, and 1 Ke1 was supposed to be defeated by 1...Kc2 2 Ne3+ Kc3 (again giving **2.1b** but this time with White to play) 3 Nd1+ Kd3 4 Nb2+ Kc2 and 5 Na4 no longer wins the e-pawn.

However, Mandler and Réti showed that even after reaching **2.1b** with White to play, it was possible to maneouvre back to it with Black to play. This is quite remarkable, because neither of White's pieces can lose a move; the knight is intrinsically unable to do so, and the king, being constrained to the first rank, is here equally inflexible. The normal triangulation manoeuvre is therefore unavailable, but though White cannot himself triangulate he can force Black to do so. Suppose 1 Ke1 Kc2 2 Ne3+ Kc3 has got us to **2.1b**; then we can continue 3 Kd1 Kd3 4 Nd5 Kd4 (4...Kc4 5 Nf4 Kc3 6 Kc1 with either 6...e3 7 Nd5+ or 6...Kc4/Kd4 7 Kd2) 5 Nb4 Kc3 (5...Kc4 6 Kd2, 5...e3 6 Nc2+) 6 Nc2 Kd3 7 Ne3 Kc3 8 Ke1 and mission accomplished.

This gives us the solution to **2.1**: **1 Ng1** (we now have the position after 1 Ke1 in the Trinks study, shifted one file to the right) **Kd2 2 Nf3+ Kd3** (**2.1b** shifted one file to the right but with White to play) **3 Ke1 Ke3 4 Ne5 Ke4** (4...Kd4 5 Ng4 Kd3 6 Kd1 with 6...f3 7 Ne5+ or 6...Kd4/Ke4 7 Ke2) **5 Nc4 Kd3** (5...Kd4 6 Ke2, 5...f3 6 Nd2+) **6 Nd2 Ke3 7 Nf3 Kd3 8 Kf1** (back to the shifted **2.1b** but with Black to play, after which we can play Trinks's finish) **Ke3 9 Ne1 Kd2 10 Nc2 Kd1 11 Nb4 Kd2 12 Nd5**.

(In *Depth and Beauty*, I pointed out that the computer gave 2 Nh3 and 4 Ng5/Nh4 as alternative winning moves but that they wasted time, and the definitive results for K + N + P v K + 2P now available have indicated further alternatives of this kind. However, valid but time-wasting alternatives for the winning side are almost inevitable in positions where the defender is wholly passive, and the question that matters is whether any of them allows White to bypass any feature which is a reason for the study's existence (here, the lose-a-move manoeuvre which gets back to the shifted **2.1b** with White to play). Some exploratory analysis here suggests that none of them would, but to give a definitive answer would require the recalculation of the table of results for K + N + P v K + 2P with the shifted **2.1b** artificially declared to be only a draw with White to play, and computing facilities capable of doing this are not currently available to me. Similar comments apply to **2.2**, where again there are many valid but time-wasting alternatives along the way. There, the key position would appear to be **2.2b**; the detailed analysis which follows on the next page would seem to preclude any significant variation in the later play.)
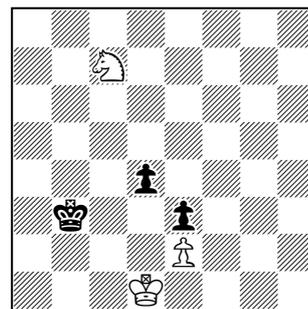
2.2 (M 6, with Artur Mandler)   **2.2a**   **2.2b**
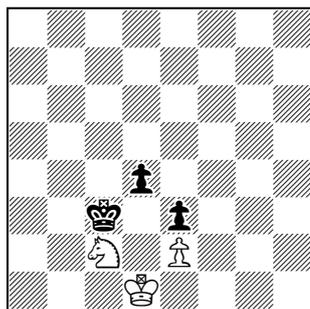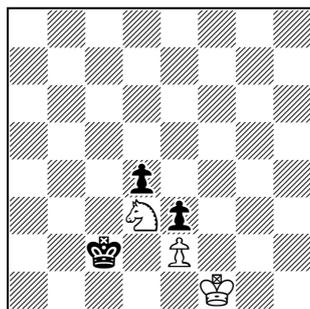
White to play and win   After 3 Ke1 Kc2 4 Kf1   Main line, after 7 Nc7

The reason that **2.1** was shifted one file to the right is that the Trinks study can also be solved by bringing the White king into play via g1/h2/g3. In **2.2**, the solution to **2.1** no longer works because the move corresponding to 11 Nb4 would take the knight off the board, and it is this alternative mechanism that must be used.

Not at once however. After 1 Ne1 Kb2 2 Nd3+ Kb3/Kc3, try 3 Ke1 Kc2 4 Kf1 (see **2.2a**). 4...Kd2 gives no trouble (5 Nf4 Kd1 6 Kg2 with Kf3 etc to follow), but if White tries to do the same after 4...Kd1, playing 5 Nf4 Kd2 6 Kg2, Black can reply 6...d3, and after 7 exd3 e2 White's pawn is lost. However, 4...Kd1 is only possible because the Black king is on c2. If in **2.2a** the Black king were on c3, White would win.

So let us put the White king back on d1, and try moving the knight away from d3 and only then taking the White king towards the east. In the position Kd1/Nf4 v Kb2, after White's Ke1, ...Kc2 fails against Nd3 (Black, being already on c2, must move away from it, and after say ...Kc3 White wins by Kf1 as we have just seen), and ...Kc1 is met by Ne6 and Nxd4. So the only correct move is ...Kc3, ready to meet Nd3 by ...Kc2. The same is true if the knight is on any other square which covers d3 and is within two moves of d4, namely b4, c5, or e5.

So if the White knight is covering d3 from b4, c5, e5, or f4, Black must reply to White's Ke1 by playing ...Kc3. We shall try to prevent this defence by reaching the position Kd1/N~ (Nb4, Nc5, Ne5, Nf4) with the Black king already on c3 and White to play.

From which of these squares, b4, c5, e5, and f4, can the knight force the Black king to play to c3? We can only do this by putting Black in zugzwang, so the knight must be guarding b3 at the instant when the Black king is on b2. So we discard b4/e5/f4, and concentrate on c5. This has taken us a further step backward (our analysis is essentially retrograde, starting from the position we want to achieve and seeing how we can force Black to let us get there): White must reach the position Kd1/Nc5 v Kb2 with Black to move.

From where could the knight have come to c5? If it is to force the king to move to b2, it must be on a square from which it controls the one important square in the Black king's field, namely c3, and on a4 it would have been controlling b2 rather than forcing the Black king to move there. So it must be on e4. The position Kd1/Ne4 v Kb3, Black to play, is won for White because Black has no reasonable move other than ...Kb2, and White's reply Nc5 gives the position of the previous paragraph.

The position Kd1/Ne4 v Kb3 is however also won if White is to play, because Kc1 forces Black to retreat and grant access to c2. So if the White knight can get to e4, he wins whether the Black king is on b2, b3, or c3.

To reach e4 from c5 is most improbable, since our whole purpose in getting to e4 is to use it as a stepping stone to c5. The square g5 is likewise not a practical choice. The knight can reach this square only from f3 or e6, and while it is doing this White will be unable to stop Black from playing ...Kc3 and ...d3.

So the practical options are d6 and f6, and the easier square to reach is f6. 1 Ne1 Kb2 2 Nd3+ Kb3 3 Nf4 Kb2 (3...Kc3 4 Ke1 Kc2 5 Nd3 Kc3 6 Kf1 etc, see **2.2a**) 4 Nd5 Kb3 5 Nf6: no, 5...Kc4! This allows 6 Kc2, but after 6... d3+ 7 exd3+ Kd4 White is powerless against the threat of ...e2. 6 Ne4 likewise is met by 6...d3.

Only d6 is left. It is easy to see that the knight must reach this square from b5, since only from here can it keep a sufficient watch on the Black king. The final question, namely how to arrive at b5, can be answered only by a detailed move-by-move analysis, which Mandler does not give, but if we work forward from the opening position we find that any play for White other than 1 Ne1 Kb2 2 Nd3+ Kc3 3 Nc1 Kb2 4 Na2 Kb1 5 Nb4 Kb2 6 Nd5 Kb3 7 Nc7 allows Black either to draw at once by ...Kc3 and ...d3 or to threaten to do so and so force White to backtrack. And from c7, the knight has access to b5.

The solution thus unfolds **1 Ne1 Kb2** (1...Kb3 2 Nd3 Kc3 transposes) **2 Nd3+ Kc3** (2...Kb1 3 Nc1 Kb2 transposes, as does 2...Kb3 3 Nf4 Kb2 4 Nd5 Kb3 5 Nc7) **3 Nc1 Kb2 4 Na2 Kb1 5 Nb4 Kb2 6 Nd5 Kb3 7 Nc7** (see **2.2b**) **Kc3** (7...Kb2 8 Nb5 and wins a pawn at once) **8 Nb5+ Kc4 9 Nd6+ Kb3/Kc3** (9...Kc5/Kd5 10 Nf7) **10 Ne4(+) Kb2 11 Nc5 Kc3 12 Ke1 Kc2/Kc4 13 Nd3 Kc3 14 Kf1 Kd2** and given is **15 Nf4 Kd1 16 Kg2** as long planned though 15 Ne5 also wins (15...Kd1 16 Nf3 and a pawn goes).
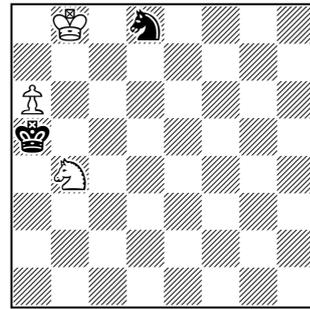
I don't know which is the more remarkable: the incredible knight journey c2-e1-d3-c1-a2-b4-d5-c7-b5-d6-e4-c5-d3 in itself, or the fact that it is required in a study with such a simple and natural starting position.

2.3 (M 8)   **2.3a**
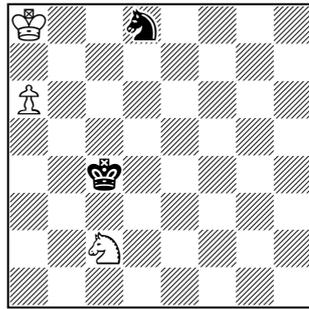
White to play and win   After 3 Kb8

The most difficult pawn for the defender to handle in endings with knight and pawn against knight is the rook's pawn. The natural move in **2.3** (composed in 1929, and first published in Mandler's 1931 book) is 1 Kb8 getting out of the pawn's way, but 1...Kb5 refutes it (2 Nb4 Nc6+ with 3 Kb7 Na5+ 4 Ka7 Kxb4 5 Kb6 Nc4+ etc or 3 Kc7 Nxb4 4 a7 Nd5+ and 5...Nb6). Correct is **1 Ka7** (blocking the pawn but threatening Kb6) **Kb5** (1...Kc5 2 Nd4 and Black has no good move) **2 Nb4 Ka5** (2...Kxb4 3 Kb6 etc) **3 Kb8** (see **2.3a**) **Nc6+** (3...Kxb4 4 Kc7 Ne6+ 5 Kb6) **4 Kb7** (4 Kc7 Nxb4 5 a7 Nd5+ and 6...Nb6) **Nd8+ 5 Kc7** (5 Kb8 Nc6+ repeats immediately, and if 5 Kc8 then 5...Nc6 forces 6 Kb7 and 6...Nd8+ again repeats) **Ne6+ 6 Kb8** (simplest though 6 Kc6 also wins, the first point at which White has had a choice) **Nc5** (6...Kb6 7 a7 Nc7 8 Nd5+) **7 a7 Nd7+ 8 Kc7** (simplest) **Nb6 9 Kb7 Kb5 10 Nd5**.