

## 4. Rook against knight and pawn on a2/a3

(from an article in *Československý šach* 1932, dedicated to master Oldřich Duras)

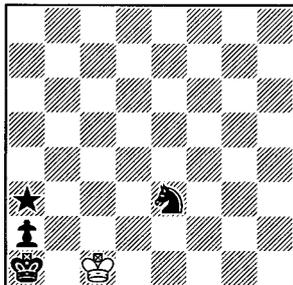
[The whole of this section of *Studie* has now been overtaken by the computer, but I think it should remain; there is considerable interest in seeing how such analyses were done before computers were available, and it provides an excellent set of puzzles for use as competition pieces or training exercises.]

In this essay, I present some studies with rook against knight and pawn which form pairs of twins or short sequences. The chapter is divided into two sections according as the the pawn is on the second or the third rank. In the essay referred to above, I gave first the studies, then the auxiliary diagrams, and finally the solutions. Here I have departed from this, giving first the auxiliary diagrams and then the studies.

### A. Pawn on a2

In this ending, Black's defence will consist in forcing stalemate (we always assume White to have the rook). If, with the Black king on a1 and the White on c1, the Black knight can play to c2 without allowing a capture giving mate by discovery, the draw is assured.

4.1 (S433)

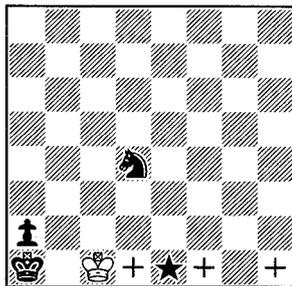


White cannot win (either side to move, wR on any square);  
same result with bN on a3

In position 4.1, the White rook can be on any square, and either side may be to move. The star indicates that the same result occurs if the Black knight is on a3.

It is easy to see that this position is drawn. If the White rook plays to the first rank, the Black knight interposes on b1 or d1, and it then returns to a3 or e3 next move. If the White rook is anywhere else, the Black knight plays to c2 and so prevents the White king from moving to this square. White can complicate matters by putting his rook on the b-file and trying to bring his king to c3 via d2, but even in this case the result is the same.

4.2 (S434)

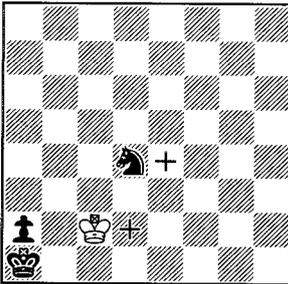


Black to move, White wins only if  
wR is on a square marked "+";  
same result with bN on e1

The square d4 plays an even more important role than a3 and e3. In diagram 4.2, the White rook must be on d1, f1, or h1 is White is to win. If the

rook is not on the first rank, Black will have an immediate draw by 1...Nc2, and even on the first rank the squares e1 and g1 are not good enough: 1...Nf3 2 Rf1 (2 Rd1 Ne1, 2 Rh1 Ng1) Nd2 3 Rd1 Nb1 and 4...Na3 will give diagram 4.1. The same happens with the knight on e1.

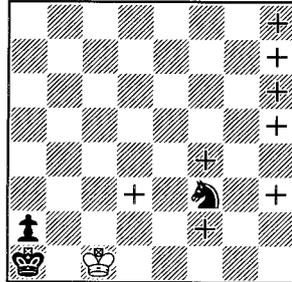
4.3 (S435)



White to move wins only if wR is on a square marked “+”

The solver will soon convince himself that White can win in position 4.3 only if his rook is on one of the two marked squares. The rook must guard both e2 and d4, the former to prevent 1 Kc3 from being met by 1...Ne2+, and the latter to prevent the knight from returning to d4 after 1...Nb5+ 2 Kb3. The square e5 is not good enough, because after 1 Kc3 Nb5+ the capture of the knight will give stalemate.

4.4 (S436)

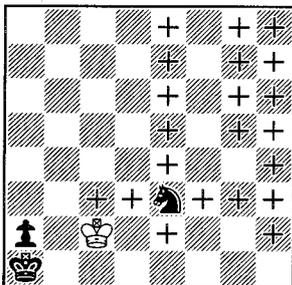


Black to move, White wins only if wR is on a square marked “+”

In diagram 4.4, Black is able to play 1...Nd4 and 1...Ne1, and we know from diagram 4.2 that White must be able to reply by playing to d1, f1, or h1. But the rook cannot already stand on one of these squares (we have seen in the analysis of 4.2 that Rd1 is met by ...Ne1, Rf1 by ...Nd2, and Rh1 by ...Ng1), nor can it stand on d5-d8 (Black draws by ...Nd4) nor on f5-f8 (Black plays 1...Nd2 threatening 2...Nb3+ and 3...Nd4+, and after 2 Kc2 Nf3 the rook cannot reach d2 or e4 as required by diagram 4.3).

The squares f4, d3, and f2 have a particular significance, in that if the rook is on one of them we have a position of reciprocal zugzwang: Black to move loses, but White to move cannot force a win.

4.5 (S437)



White to move wins only if wR is on a square marked “+”

Diagram 4.5 demonstrates that the White rook is badly placed on the d or f file. White wins only if his rook is on one of the squares marked “+”. With the rook on d4, neither 1 Kb3 nor 1 Kc3 suffices to win, because there follows 1...Kb1 2 Rd3 and either 2...a1Q or 2...a1N+.

[For once, I found myself in need of a little further explanation. 1 Kc1 concedes the draw at once (see 4.1), so White must play 1 Kb3 or 1 Kc3. Black naturally replies 1...Kb1, and White must be able to respond either by capturing the knight or by a first-rank check.]

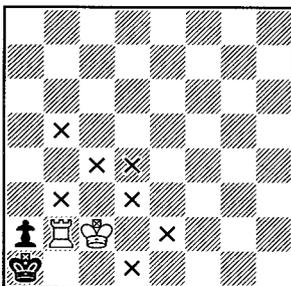
unless the Black knight is on one of the squares marked with a cross. We have already met the case d4 in diagram 4.3, where square b2 is not marked with a plus sign, and in fact the draw is immediate (1 Kc3 Nb5+, 1 Kc1 Nb3+). If Black merely threatens to check on d4, for example if the knight is on f3, White wins by 1 Kc3 Nd4 2 Rd2, but of course this option is not available if the knight is on b5 or e2.

Black will also draw if the knight is on c4, d3, or d1. However, a4 is not good enough, because Black will need three moves to give check and in the meantime the White rook can transfer itself to the h-file and threaten mate, for example 1 Rb8 Nb2 2 Rh8 Nd1 3 Re8 Ne3+ 4 Kb3 etc.

[Mandler’s diagram omits b3. He treats this square the same as f3, pointing out that 1 Kc3 Nd4 2 Rd2 wins (which it does) but overlooking the drawing move 1...Nc1. Now 2 Rd2 can be met by 2...Kb1 without allowing a bottom rank mate, and White must return to b2 (or play Rxa2) if he is not actually to lose.]

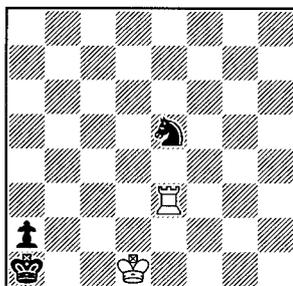
This is perhaps enough for us to solve the studies in diagrams 4.7-4.10. The solutions will be found on page 93.

4.6 (S438)



White to move wins unless bN is on a square marked “x”

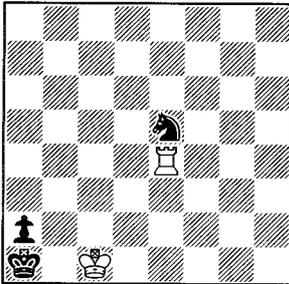
4.7 (S439-40)



White to move and win (a) as set, (b) wR on e4

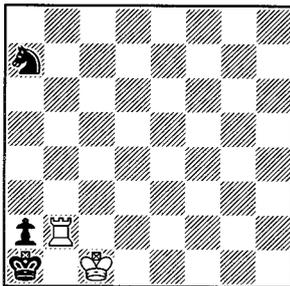
In Diagram 4.6, White to move wins

4.8 (S441-2)



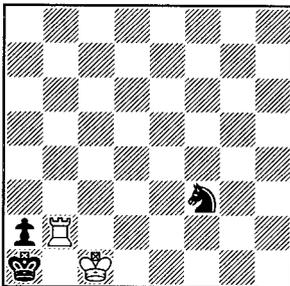
White to move and win  
(a) as set, (b) wR on g7

4.9 (S443-5)



White to move and win  
(a) as set, (b-c) bN on e7/f2

4.10 (S446-9)

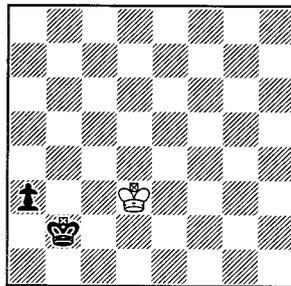


White to move and win  
(a) as set, (b-d) wR on b3/b4/b5

B. Pawn on a3

Endings in which the Black pawn is on a3 are rather more difficult, because they sometimes come down to R v N with no pawn and the solver must know the theory of this ending at least in its essentials. We also need to look at the R v P endings which may arise after a sacrifice of the knight, and the first two of our preliminary diagrams will address these.

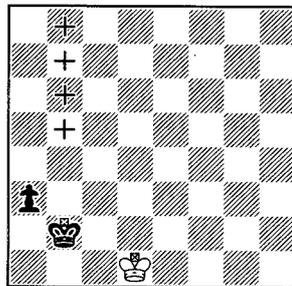
4.11 (S450)



R v P: White to move cannot win,  
wherever the rook may be

This position is always drawn. If for example 1 Rg2+ then 1...Kb3, and after 2 Rg8 then 2...Kb2! 3 Rb8+ Kc1 with a draw.

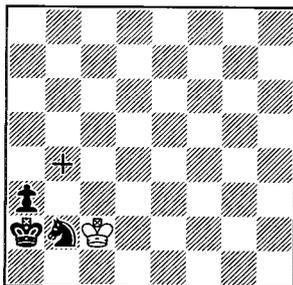
4.12 (S451)



R v P: Black to move, White wins  
only if wR is on a square marked "+"

If the rook is on b4, Black plays 1...Kc3 2 Ra4 Kb2 (not 2...Kb3?) and White has no winning continuation. If the rook is on b5 or any higher square, White wins easily (1...Kc3 2 Kc1 a2 3 Ra5 Kb3 4 Ra6 etc).

4.13 (S452)

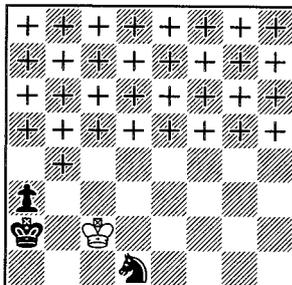


Black to move, White wins only if wR is on the square marked “+”

This is one of the most important positions. White wins only if his rook is on b4. On 1...Ka1 there now follows 2 Kb3 Nd3 3 Rd4 Nc1+ (3...Nc5+ 4 Kxa3) 4 Kc2 Ne2 5 Rd2 and wins.

If the rook is on d4, Black draws by 1...Nd1, because the White king cannot take the knight on account of 2...Kb2 (see 4.12) and 2 Re4 is met by 2...Nf2.

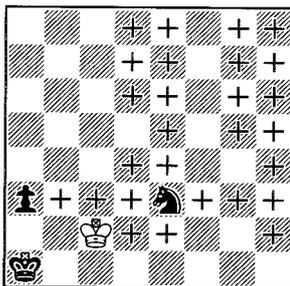
4.14 (S453)



White to move wins only if wR is on a square marked “+”

This diagram illustrates one of the consequences of diagram 4.12. Unless the White rook is already on the b-file, Black will answer White's 1 Kxd1 by 1...Kb2, and White must be able to reply by a check on b5 or above. A check on b4 is not sufficient.

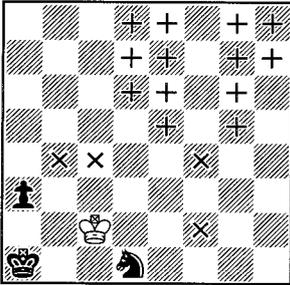
4.15 (S454)



White to move wins only if wR is on a square marked “+”

If we compare this position with that of diagram 4.5, where the pawn is on a2, we see that it is more favourable for White, who can win not only when the rook is one of the squares previously marked but also on b3 and on most of the d-file (apart from d5 and the impossible d1). For example, rook on d4: 1 Kb3 Kb1 2 Rd3 a2 3 Rxe3 etc. But the rook is badly placed on the f-file (apart from on f3, whence it can capture the knight), because 1 Kb3 Kb1 2 Rf3 can be met by 2...Nd1.

4.16 (S455)



Black to move, White wins only if  
wR is on a square marked “+”

White to move wins unless  
wR is on a square marked “x”

Two moves come into particular consideration for Black: 1...Ne3+ and 1...Ka2. The first leads to the preceding diagram, the second to diagram 4.14. White can hope to win only if the rook stands on a square which is marked “+” in both these diagrams. But if we compare the three diagrams, we see that the present diagram has no “+” on h5 and h6. This is because Black has another move, 1...Nf2, which holds the draw if the rook is on one of these two squares. After 1...Nf2 2 Rh2 Ng4 3 Rh4 Nf2 4 Rf4 Nd1 White has no good continuation. The king cannot take the knight (see 4.12), while rook moves to d4, e4, h4, f1, or f3 allow 5...Ka2 (see 4.14) and other moves are met by 5...Ne3+ (see 4.15).

After 1...Nf2, if the rook is on h6, the try 2 Rf6 is met by 2...Ng4 (3 Rf4 Ne3+, or 3 Rg6 Ne5 4 Rg5/Re6 Nf3).

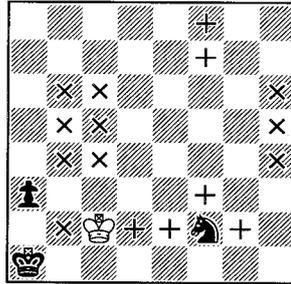
If the rook is on h5 and White tries 2 Rf5, Black draws by 2...Nd3, because the king cannot capture (see 4.11), the rook cannot attack the knight (3 Rd5 Nb4+, 3 Rf3 Ne1+), and any other rook move is met by 3...Ka2.

If the rook is on h7 or h8, White meets 1...Nf2 by 2 Rf7 (Rf8).

[The computer pedantically adds a

cross on b2, but this square is of no practical importance (a rook here would be attacked by two Black men, so why didn't Black use one of them to capture it last move?) and Mandler obviously thought it irrelevant. The same is true of some later diagrams.]

4.17 (S456)



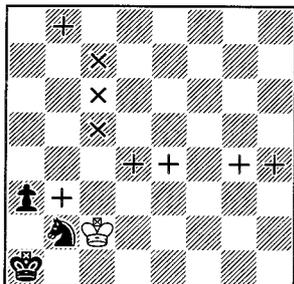
Black to move, White wins only if  
wR is on a square marked “+”

White to move wins unless  
wR is on a square marked “x”

The signs on the f-file require little explanation. We have just seen that White wins against a knight on f2 by playing his rook to f7 or f8, and on f3 the rook threatens immediate mate. The reader can likewise easily convince himself that White wins if the rook is on d2, e2, or g2. We saw in the analysis of the last diagram that h2 was a bad square (in the line 1...Nf2 2 Rh2 Ng4 etc).

[The computer adds a trivial “+” on f1, again doubtless omitted by Mandler on the grounds that it is of no practical importance.]

4.18 (S457)

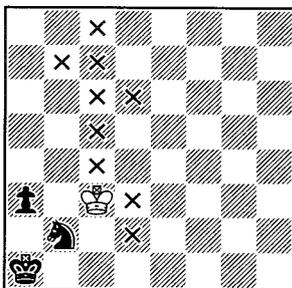


Black to move, White wins only if  
wR is on a square marked “+”  
White to move wins unless  
wR is on a square marked “x”

Black threatens 1...Ka2, and we know from diagram 4.13 that White must be able to meet this by playing to b4. The square f4 is insufficient on account of 1...Nd1 (see 4.16). If the rook is on b5, Black escapes by playing 1...Nd3 (see 4.24 later), and if it is on b6 or b7 Black has 1...Nc4 (see 4.21, likewise later).

[Mandler presumably regarded the crosses as self-explanatory. There are none on the f-file because White to move would play 1 Kb3 with a quick mate.]

4.19 (S458)



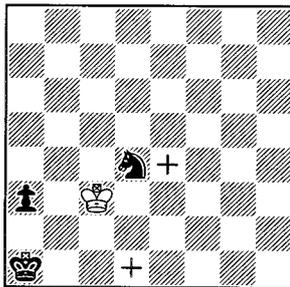
Black to move can always draw  
White to move wins unless  
wR is on a square marked “x”

There are no “+” signs on this diagram

because Black to move can draw irrespective of the position of the rook.

The crosses on b7, d6, and d2 deserve particular attention. If the rook is on b7, Black meets 1 Kb3 by 1...Nd3 (2 Rd7 Nc5+) and 1 Kc2 by 1...Nc4 (see 4.21 below). If it is on d6 or d2, the line 1 Kb3 Kb1 2 Kxa3 is defeated by 2...Nc4+.

4.20 (S459)

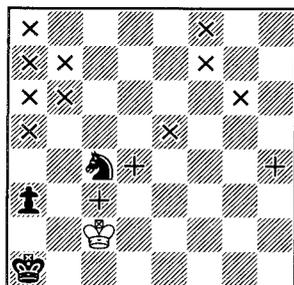


Black to move, White wins only if  
wR is on a square marked “+”

The plus sign on d1 is only for completeness. After 1..Ka2, White of course takes with the rook and not the king. Otherwise we have a position reminding us of diagram 4.3 after 1 Kc3, and the logic is the same: the rook must cover both e2 and d4, so as to prevent an immediate 1...Ne2 and also a return to d4 after 1...Nb5+ 2 Kb3. However, there is a difference. In diagram 4.3, both e4 and d2 were suitable squares for the rook. With the pawn on a3, only e4 works. If the rook is on d2, Black can play 1...Ne6 without allowing immediate mate, and he will be able to meet 2 Kb3 with 2...Nc5+.

[This is the first serious error in Mandler's analysis, and I have had to alter his text. He puts a plus sign on d2 as well, overlooking 1...Ne6.]

4.21 (S460)



Black to move, White wins only if  
wR is on a square marked “+”  
White to move wins unless  
wR is on a square marked “×”

We have referred to this diagram in the analysis of positions 4.18 and 4.19. The square f4 is not marked with a plus sign on account of 1...Ne3+ (see 4.15) and e4 on account of 1...Nd6 2 Rd4 Nf5 3 Rf4 Ne3+.

Now to the crosses. If the rook is on b8, White wins by 1 Kb3. This fails with the rook on b7 (1...Na5+). The draw with the rook on b6 follows from diagram 4.15, since if the rook attacks the knight by 1 Rb4 or 1 Rc6 Black will reply 1...Ne3+.

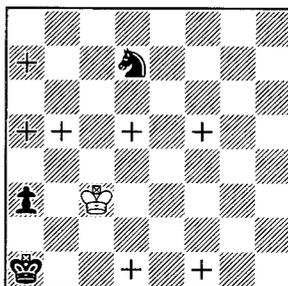
With wRg8, White plays 1 Kc3 Na5 2 Kb4 Kb2 3 Rg2+ Kb1 4 Kxa3 and wins with R v N, but with wRf8 the corresponding line is only drawn (play continues 4...Nc4+ 5 Kb3 Ne3 and the mating square is covered). We also have 1 Kb3 Nd2+ 2 Kc3 Ne4+/Kb1, and 1 Rf4 Ne3+ (see 4.15).

With wRf7, 1 Kc3 is no longer defeated by 1...Na5 (2 Rd7 wins), but 1...Ne3 2 Rf3 Nd1+ 3 Kc2 leads to diagram 4.16. With wRg6, 1 Kc3 is met by 1...Ne5.

[This position is more difficult than Mandler thought. He omits the crosses on f8 and g6, and less seriously those on the a-file and e5 and the plus on c3, and though I have tried to alter his text to

highlight the essentials I do not claim to have provided a full treatment.]

4.22 (S461)

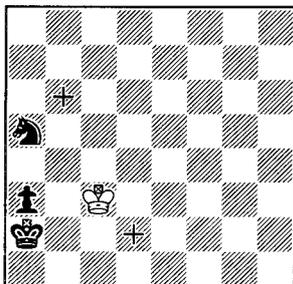


Black to move, White wins only if  
wR is on a square marked “+”

If the rook is on the d-file and not on a marked square, Black will draw by playing 1...Nc5, since if White then attacks the knight Black will play 2...Kb1 and the rook will be unable to take it.

[Mandler omits the plus signs on a5, b5, and f5, where White wins even though the knight is not under immediate attack, and also that on f1. With the rook on g5 or h5, Black draws by playing 1...Kb1 and if 2 Kb3 then 2...Kc1, but if it is on f5 White can continue 3 Kxa3 and then round up the knight. With the the rook on a5, b5, or f1, 1...Kb1 is either illegal or useless, and if Black plays 1...Ka2 White can continue 2 Rf5 with a difficult win.]

4.23 (S462)



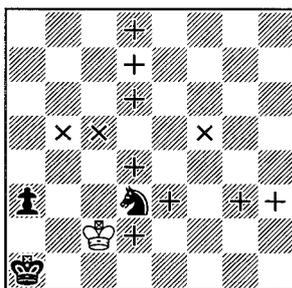
Black to move, White wins only if wR is on a square marked “+”

This position is won for White only if the rook is on d2 or b6. If it is on say g2 instead of d2, Black draws by 1...Kb1 2 Rg5 Nc6 3 Kb3 Nd4+. This check is not available if the rook is on the d-file.

The square b6 is likewise good for White. Black must play 1...Ka1, and there follows 2 Rd6 Kb1 3 Rd5 Nc6/Nb7 4 Kb3 etc.

However, if the rook is on b6 with White to play, he must abandon his favourable position and there is no win.

4.24 (S463)



Black to move, White wins only if wR is on a square marked “+”

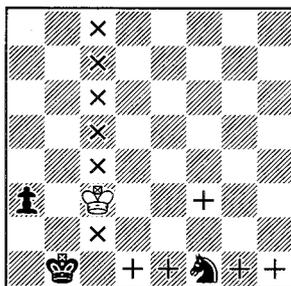
White to move wins unless wR is on a square marked “x”

With Black to move, 1...Nc5 does not help, because in contrast to diagram 4.22

the White king is on c2.

With White to move and the rook on b5, 1 Rb3 is met by 1...Ne1+. If it is on c5, 1 Rc3 fails against 1...Nb4+.

4.25 (S464)



Black to move, White wins only if wR is on a square marked “+”

White to move wins unless wR is on a square marked “x”

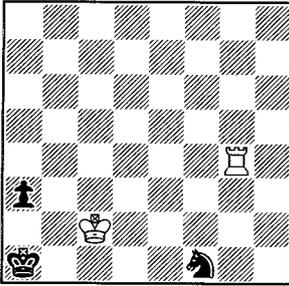
If the rook is giving check from e1, the solution is easy: 1...Ka2 2 Re2+ Kb1 3 Kb3. The play is similar if the rook is on g1, but the solver must be aware that he cannot win without allowing Black to promote to a second knight: 1...Ka2 2 Rg2+ Kb1 3 Kb3 a2 4 Rg1 a1N+ 5 Kc3 etc.

If the rook is on f3, White wins by 1...Nd2 2 Re3 Nf1 3 Re1+ and as above, or 1...Nh2 2 Rg3 Nf1 3 Rg1 etc.

If the rook is giving check from d1, the procedure is 1...Ka2 2 Rd3 Kb1 3 Rf3 etc. The win with the rook on h1 is analogous: 1...Ka2 2 Rh3 Kb1 3 Rf3.

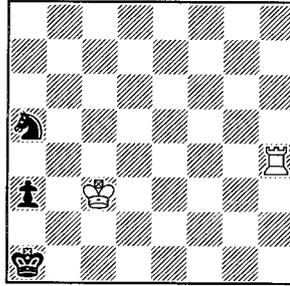
This preparation will simplify the analysis of the following diagrams. The solutions are on pages 93-4.

4.26 (S465-7)



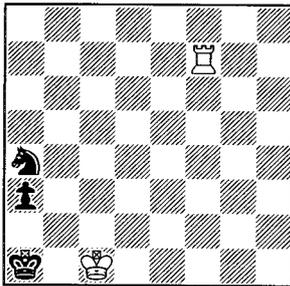
White to move and win  
(a) as set, (b-c) wR on g5/h5

4.29 (S477-8, version)



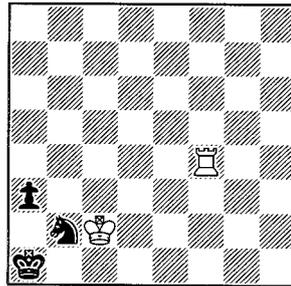
Black to move and draw  
(a) as set, (b) wR on h6

4.27 (S471-2)



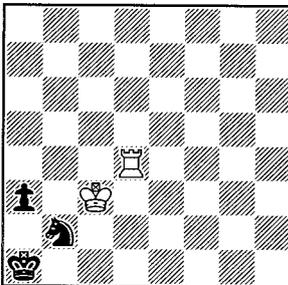
White to move and win  
(a) as set, (b) wR on f4

4.30 (S479-82)



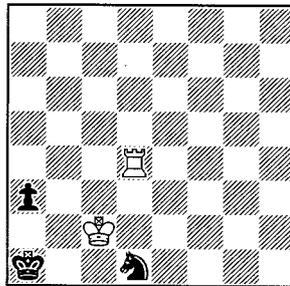
Black to move and draw  
(a) as set, (b-d) wR on b5/b7/g5

4.28 (S497-8)



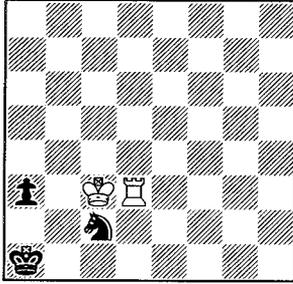
White to move and win  
(a) as set, (b) wR on d7

4.31 (S483-5)



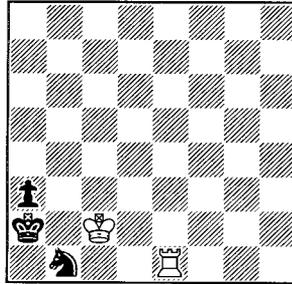
Black to move and draw  
(a) as set, (b-c) wR on f6/h6

4.32 (S486-8)



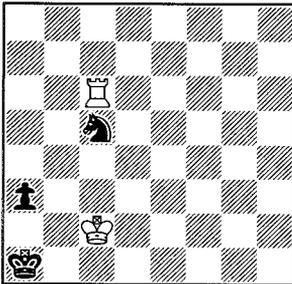
Black to move and draw  
(a) as set, (b-c) wR on g2/f5

4.35 (S493-4)



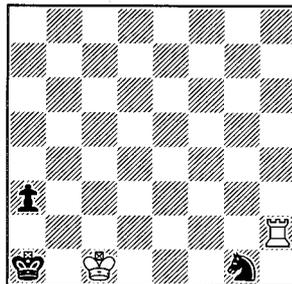
Black to move and draw  
(a) as set, (b) wR on d1

4.33 (S489-90)



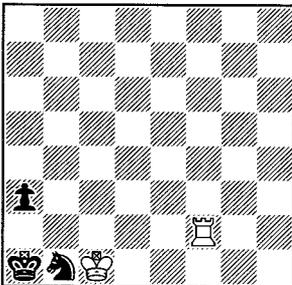
Black to move and draw  
(a) as set, (b) wR on c8

4.36 (S495-6)



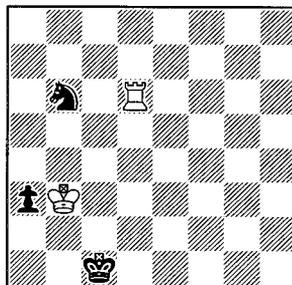
Black to move and draw  
(a) as set, (b) wR on d2

4.34 (S491-2)



Black to move and draw  
(a) as set, (b) wR on h2

4.37 (S499-500)



Black to move and draw  
(a) as set, (b) wR on e6

4.7. Only two moves come into consideration: 1 Kc2 and 1 Kc1. Correct in (a) is **1 Kc2 Nf3 2 Re4** (Black is going to play 2...Nd4+, so White must play to one of the squares marked "+" in 4.3) **Nd4+ 3 Kc3 Nb5+ 4 Kb3** and wins. If instead Black plays 1...Nd3, there are several ways to win, for example 2 Re4 Nb4+ 3 Kb3 Nd3 4 Rd4.

1 Kc1? is not defeated by 1...Nf3? hoping for 2...Nd4 reaching 4.2, because White has 2 Rd3 winning (see 4.4). Instead, Black must play 1...Nc4/Ng4 2 R-- Na3/Ne3, giving 4.1.

In (b), 1 Kc2? fails to 1...Nf3 (see 4.3). As we have seen, this position (wKc2, wRe4, bNf3) is a position of reciprocal zugzwang: Black to move would lose, but White to move must weaken his position. Correct is **1 Kc1 Nf3 2 Kc2** and it is Black to move, or 1...Nc4 2 Kc2 Ne3+ 3 Kb3.

4.8. In (a), **1 Re2 Nf3 2 Rf2** (see 4.4), or 1...Nd3+ 2 Kc2 Nb4+ 3 Kb3 Nd3 4 Rd2. We know from part (b) of the preceding study that 1 Kc2 does not work and from part (a) that 1 Re3 is met by 1...Nc4, while 4.5 helps to show that 1 Rd4 is not correct: Black will continue 1...Nc4 (threat 2...Na3, 4.1) 2 Kc2 Ne3+ etc.

In (b), **1 Re7 Nd3+/Nc4 2 Kc2**; 1 Rg1? Nf3! (see 4.4). 1 Kc2? Nf3! and the rook cannot reach e4 or d2 (see 4.3), while on 2 Kc1 Black will play 2...Ne1 or 2...Nd4 (see 4.2).

[I have presented the four studies of 4.7 and 4.8 as two pairs to make the diagramming easier. Mandler, who gives each study a separate diagram, presents them as a set of four, which emphasizes the link between 4.7 (b) and 4.8 (a).]

4.9. In (a), Black threatens 1...Nb5 and 2...Na3, which will draw according to diagram 4.1. To avoid this draw, White must play Kc2 at his first or second move. But 1 Kc2 Nb5 gives

diagram 4.6, and again White cannot win. So White must play Kc2 at move 2, and Black will be able to reply by giving check on d4. So White must put his rook on one of the squares shown in diagram 4.3, and e4 is not within range; so the solution is **1 Rd2 Nb5 2 Kc2 Nd4+ 3 Kc3** etc.

In (b), 1 Rd2 fails against 1...Nf5 2 Kc2 Ne3+ (see 4.5), and 1 Rh2 against 1...Nf5 2 Kc2 Nd4+ (see 4.3). However, White now has **1 Kc2**, since the knight cannot reach any of the squares marked in diagram 4.6.

In (c), 1 Kc2 is met by 1...Nd1/Nd3 (see 4.6), while 1 Rd2 Nd1/Ng4 2 Kc2 Ne3+ puts us into diagram 4.5. Correct is **1 Re2**.

4.10. These four studies can be solved very easily by considering diagram 4.4, because in each case only one of the marked squares can be reached. In (a), therefore, **1 Rf2**; in (b), **1 Rd3**; in (c), **1 Rf4**. In (d), **1 Rh5 Nh2 2 Kc2 Nf1 3 Kb3 Nd2+ (3...Kb1 4 Rh1) 4 Kc3 Ne4+ 5 Kc2** etc; not 1 Rf5? Nd2 2 Kc2 Nf3 3 Rf4 Nd4+ and draws.

[Again, Mandler presents the seven studies of 4.9 and 4.10 as a single set. The four studies of 4.10 would be a very interesting group to set for solution without Mandler's preliminary analysis; I wonder how many players, even of master strength, would get them all right first time.]

4.26. In (a), **1 Kb3 Nd2+ 2 Kc3** and now 2...Nf1 3 Rg1 and wins (see 4.25) or 2...Nf3 3 Rf4 (not 3 Rg3); if 2...Nb1+ then 3 Kc2 wins, for example 3...a2 4 Rg2 Nd2 5 Kc3 Nb1+/Ne4+ 6 Kb3. Not 1 Kc3? Ne3! 2 Re4/Rg3 Nd1+ (see 4.16).

In (b), **1 Kc3 Kb1 2 Rg1** (see 4.25); if 1...Ne3 then 2 Kb3. Not 1 Kb3? Nd2+!

In (c), **1 Rh3 Ka2 2 Kc3 Kb1 3 Rf3** etc (see 4.25). Not 1 Kc3? Ng3

2 Rg5/Rh3 Ne2+ 3 Kb3 Nd4+ (see 4.20).

[Mandler has the rook on h6 in the diagram of (c) but "2 Rg5" in the text. The solution is the same with the rook on h6, but the twinning g4-g5-h5 is neater and I have assumed that the misprint is in the diagram.]

4.27. In (a), 1 Ra7 Nc3 2 Kc2 Nb5 3 Rd7 Ka2 4 Rd2 and either 4...Na7 5 Kc3+ Kb1 6 Kb3 Kc1 7 Rd5 or 4...Nc7 5 Kc3+ Kb1 6 Kb3 etc.

In (b), 1 Kc2 Nc5 2 Rc4 (not 2 Rf5, see 4.24).

4.28. In (a), 1 Kc2 Ka2 2 Rb4 (see 4.13 etc); 1 Kb3? Kb1! 2 Kxa3 Kc2.

In (b), 1 Kb3 Kb1 2 Kxa3; 1 Kc2? Ka2! (see 4.13).

[Mandler gives this towards the end of the "Black to play and draw" group, but it seems more conveniently placed here and I have taken the liberty of moving it. Part (b) seems to work just as well with the rook on d5 instead of d7.]

4.29. In (a), 1...Ka2 and draws because White can reach neither d2 nor b6 (see 4.23). 1...Kb1? 2 Rb4+! Ka2 (2...Kc1 3 Ra4) 3 Rb6 etc.

In (b), 1...Kb1 2 Rb6+ Ka2/Kc1. 1...Ka2? 2 Rb6.

[Mandler has the rook on g4 and g6, but this allows an alternative refutation of 1...Ka2 in (b): 2 Kb4 Nb7 3 Rg2+ Kb1 4 Kxa3 and the knight falls in 13 more moves.]

4.30. In (a), 1...Nd1 (see 4.16).

In (b), 1...Nd3 (see 4.24).

In (c), 1...Nc4 (see 4.21).

In (d), 1...Ka2 (see 4.13).

4.31. In (a), 1...Ka2 (see 4.14).

In (b), 1...Ne3+ (see 4.15).

In (c), 1...Nf2 (see 4.26).

4.32. In (a), 1...Ne1 2 Re3 Ng2 3 Re2 Nf4 4 Re4 Nd3 and either 5...Rd4/Re3 Ne5 etc (see note to diagram 4.22) or 5...Kxd3 Kb2 (see 4.11). If 4 Rf2 then 4...Ne6 5 Rf6 Nc5; if 3 Rg3 then 3...Nf4 4 Kb3 (4 Rf3/Rg4 Ne2+) Ne2 5 Rg2 (5 Re3 Nd4+ 6 Kc3 Kb1, see 4.20) Nd4+ 6 Kxa3 Kb1.

In (b), 1...Ne3 (1...Ne1? 2 Re2!) 2 Re2/Rg3 Nd1+ 3 Kc2 Ka2 (see 4.16).

In (c), 1...Nd4 (see 4.20); 1...Ne3? 2 Re5! Nd1+ 3 Kc2 (see 4.16).

4.33. In (a), 1...Ne4 2 Re6 (2 Rc4 Nf2 3 Rf4 Nd1, see 4.16) Ng5 (2...Nc5? 3 Re5 Nd3 4 Re3, see 4.24) 3 Re5/Rg6 Nf3 and draws.

In (b), 1...Ne6. 1...Ne4? 2 Re8.

4.34. In (a), 1...Nc3 2 Kc2 Nd1 (see 4.16); 1...Nd2? 2 Kc2 and wins, because the knight cannot use f3 to reach d4.

In (b), 1...Nd2 2 Kc2 Nf3. Not 1...Nc3 on account of 2 Kc2 Nd1 3 Rh7/Rh8 (see 4.16).

["The White rook can also stand on g2", writes Mandler about (b), and I think I would put it there even though it gives White a choice of four moves, 3...Rg5/g6/g7/g8, in refuting 1...Nc3.]

4.35. In (a), 1...Nd2. 1...Nc3? 2 Re5! Na4 3 Rb5 Nb2 4 Rb4 (see 4.13).

In (b), 1...Nc3, because White does not have e5 at his disposal.

4.36. In (a), 1...Ne2+ 2 Kc2 Nd4+ 3 Kc3 Kb1; 1...Nf3? 2 Rh3!

In (b), 1...Nf3! 2 Rd3 a2! and White has no good move.

4.37. In (a), 1...Na4.

In (b), 1...Na4 2 Kxa4 a2 3 Kb3 and wins; 1...Nd7!