2. Rook against knight

[This endgame is classic territory, having first been investigated in the ninth century. The analysis was finally completed by computer in 1970, but it took a while for the news to filter through the chess community and there is no evidence that Mandler was aware of it. And even in the presence of the definitive analysis now available, I think Mandler's studies remain of interest. The computer merely divides positions into two classes, won and not won; the studies probe the boundaries, and throw light on why a certain position ends up on one side of the fence while an almost identical position finds itself on the other.]

A surprisingly quick victory

2.1 (S336)
Revue FIDE 1955
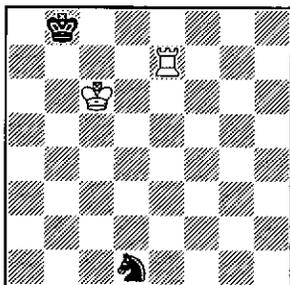
How quickly can White win?

This is neither a problem nor a study, it is something between the two. The stipulation can be specified more precisely: Within three moves, White must achieve a position where either mate or capture of the knight will follow next move. Strictly speaking, such compositions belong to fairy chess, but the present one will serve as an introduction to the less easy positions in the ending R v N.

In this ending, the knight is in greatest danger when it ventures too close to the enemy king, or when it finds itself too far away from its own king. In our example here, it is far from its own king, without protection. A mere two moves, **1 Kb6 Kc8 2 Re2**, now leave Black with no good move. But as a problem "win in *n* moves" this is strictly speaking a three-mover, because if Black plays 2...Nc3 the knight is not lost at once; it is captured only after 3 Rc2.

[The term "fairy chess" was coined by Henry Tate of Melbourne "for all that immense range of work which stands outside, in some point or other, the orthodox channels of Caissic ingenuity" (T. R. Dawson, *The Chess Amateur*, December 1918, p 85). In calling "win within *n* moves" a "fairy chess" stipulation, Mandler was taking rather a strict view, because the stipulation was orthodox enough in the early days of chess and may yet become so again. It certainly seems appropriate to compositions such as **1.13**, where "reach a simply won position within 9 moves" retains the author's intended solution while eliminating a long-winded alternative demonstrable only with the aid of a computer.]

A beautiful and theoretically very important study, though only the first move is mine

2.2 (S337)
Československý šach 1933
(after F. Amelung)
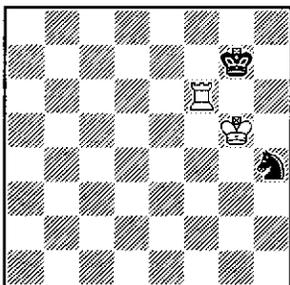
White to move and win

In 1900, F. Amelung published the following study: White Kf4, Rd6 (2), Black Kg7, Nh4 (2), White to move and win. I don't know where it appeared. I have given the study a different first move, but everything else remains the same. In the diagram, the solver must look several moves ahead in order to find the right move. The solution unfolds **1 Rd6 Nf3+ 2 Kg4** (2 Kf4? Nh4 3 Kg4 Ng6 and draws) **Ne5+ 3 Kf5 Nf7** (for 3...Nc4 see below) **4 Rd7** (see **2.2a**) **Kg8 5 Kf6** and wins. **2.2a** is a fundamental position in this ending, and we shall frequently encounter it.
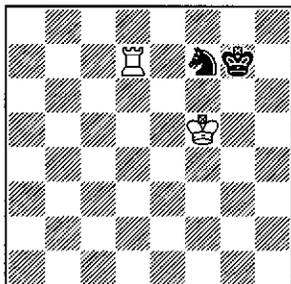
2.2a - Black to move, White wins.

Black can hold out longer if he keeps his knight further away from the White king. If we play **3...Nc4** instead of 3...Nf7, White must avoid 4 Re6 on account of **4...Kf7** (another important position) **5 Rc6 Ne3+ 6 Kf4 Nd5+ 7 Ke5 Ne7**, for after **8 Rc7 Kf8** there is no win. After 8 Rc7 the men are placed as in **2.2a**, but the position has been shifted one file to the left. So we see that "Amelung's position" - for thus we would like to call **2.2a** - cannot be shifted either to the left or downwards without forfeiting the win.

Instead of 4 Re6, White must answer 3...Nc4 by **4 Rd4 Ne3+ 5 Kf4 Nc2 6 Rc4 Na3 7 Rc5 Kf6 8 Ke4 Ke6 9 Kd3**, and now he does win.

[This is a study where Mandler does not highlight a main line, and it is clear that he regarded the draw after 3...Nc4 4 Re6 as just as important as the two winning lines. In discussing **2.2a**, Mandler writes "Kf8" and not Kg8, but Kf8 loses very easily whereas Kg8 parallels the move which draws when the position is shifted one file to the left, and I am sure the latter is what was intended.

We may also notice that Mandler gives 2 Kf4 an unequivocal question mark, even though White can meet 2...Nh4 by 3 Kg5 and have another bite at the cherry. In the analysis of a study to win, a move which allows the defender to regain a previous position is regarded as faulty, even if a player in a practical game would have an opportunity to try again. It is as if the "three repetitions" rule did not exist, and *any* repetition of a previous position allowed the weaker side to claim a draw.

As for the origin of the Amelung position, Ken Whyld tells me that *Deutsche Schachzeitung* ran a series by Amelung on R v N in 1900. The position here was quoted on page 138, with a comment that he had analysed it in "Balt. Schachbl" (presumably *Baltische Schachblätter*), number 6, page 223.]

One apparently insignificant square makes all the difference

*2.3 (S338)
Oesterreichische Schachrundschau
1924
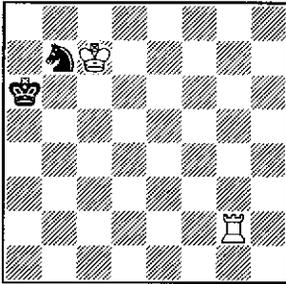
White to move and win

In this position, if the rook were on h2 instead of g2 there would be no win. **1 Rg6+** (the moves 1 Ra2+ Na5 would lead to a fundamental and well-known draw, in which the Black pieces support each other and prevent the approach of the White king) **Ka7 2 Kc6 Nd8+**. If 2...Ka6 then White waits with 3 Rh6 or 3 Rf6, and we have a win known to Amelung: 3 Rh6 Na5+ 4 Kc5+ Ka7 5 Kb5 Nb7, and we have reached the winning position of al-Adli from the year 1257(!). **3 Kd6!!** Here we see why there would be no win with the rook on the h-file: it would now stand on h6, and Black could capture it by ...Nf7+. **3...Nb7+ 4 Kd5!** This and the preceding move are better explained by variations than by words. **4...Na5 5 Kc5 Nb7+ 6 Kb5 Kb8 7 Kc6 Nd8+ 8 Kd7 Nb7 9 Rg5 Ka7 10 Kc8** and White wins.

[Mandler now gives a detailed analysis covering over 20 lines, which we reproduce with definitive computer commentary in Appendix B. And the "al-Adli" position is even older than Mandler thought. H. J. R. Murray dates al-Adli's chess activity to the middle of the ninth century, and cites a report that

the position had actually occurred in a game played by Rabrab, who was active in 819 (*A history of chess*, Oxford, 1913, pp 197 and 307).]

An unlikely opening move

2.4 (S339)
28. říjen 1926
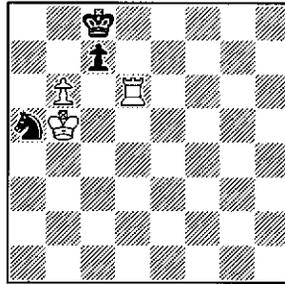
White to move and win

The first move is **1 Rd2**. Clearly some solvers would exclude this move from consideration, since the continuation 1...cxb6 2 Kxb6 Nc4+ loses the rook. But a knowledge of the preceding study, where in essence we have the same position reflected about the long diagonal, simplifies the solution for us. After **1...cxb6** (1...Nb3 would be met by 2 Rd3 Nc1 3 Re3 etc) there follows **2 Rc2+ Kb7** (2...Kb8 3 Kxb6 Nb7 4 Kc6 Na5+ 5 Kc5 etc as in the preceding study) **3 Rc3**. During the solution of the preceding study, we saw that the rook could not start on the h-file, but we might have added that it could have started on the f-file instead of the g-file, and the f- and g-files correspond here to the second and third ranks. The move 3 Rc3 forces the Black king to retreat. **3...Kb8 4 Kxb6 Nb7 5 Kc6 Na5+ 6 Kc5** etc as in the preceding study.

White cannot start 1 Rd3 on account of 1...cxb6 2 Rc3+ Kb7, after which he has no waiting move.

[This study appears in Harold van der

Heijden's "Endgame study database 2000" with a note claiming an alternative win by 1 Rd1 cxb6 2 Rc1+ Kb7 3 Rc3 leading back into the main line, but this is quite false: 2...Kb8! holds the draw.]

Another variation on the same theme

2.5 (S340)
Československý šach 1933
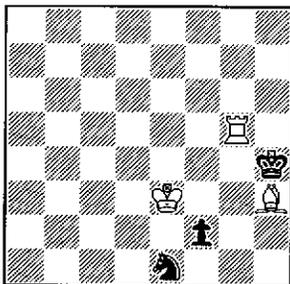
White to move and win

1 Rb5 Kxh3 2 Kxf2 Ng2 3 Rb3+ etc. The position is now as after White's first move in the *Oesterreichische Schachrundschau* 1924 study, rotated through 180 degrees.

Everything seems obvious ...

2.6 (S341)
A universally known theoretical position
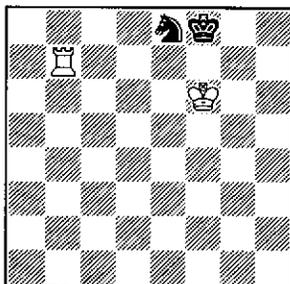
White to move, Black draws

Those familiar with the ending R v N will see at a glance that this position is drawn. If White plays **1 Ke6**, there follows **1...Ng7+ 2 Kf6 Ne8+ 3 Kg6 Kg8 4 Rf7 Nd6 5 Rd7 Ne8** and so on.

Everything is obvious, and the question of space plays no role here. At least this would be the judgement of the majority of solvers.

... but even in the ending R v N, space plays a major role

2.7 (S342)
Československý šach 1933
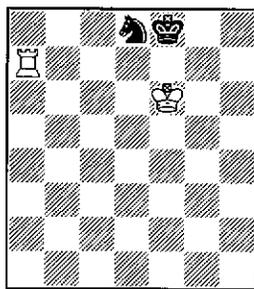
b   c   d   e   f   g   h

Here we have modified the preceding diagram by removing the a-file. This slight narrowing of the available space is sufficient to convert a clearly drawn position into a win. So even in the ending R v N, the question of space plays a significant role.

The present study does not use the normal board, and so belongs strictly to the realm of fairy chess. However, it is very useful for the understanding of the ending R v N. We cannot solve it without some fundamental theoretical analysis, and in particular it is necessary to be familiar with the *Oesterreichische Schachrundschau* 1924 study.

After **1 Kg6 Nd6** (1...Kg8 2 Rd7 Kf8 3 Rf7+ leads to the same position) **2 Rd7 Ne8** (if 2...Nb5 then 3 Kf6 Ke8 4 Ke6 and either 4...Nc3 5 Rc7 or 4...Kf8

5 Rd8+ Kg7 6 Rd3, while if 2...Nc4 then
3 Rd4 Nb6 4 Rd6 Nc4 5 Re6) **3 Rf7+
Kg8** the rook must quit the seventh rank,
otherwise the Black king will be able to
return to the f-file. Which square on the
f-file should the rook choose? We know
from the preceding studies that only f2
and f3 come into consideration. Here we
play **4 Rf3**. As regards 4 Rf2, we content
ourselves with the observation that after
4...Nd6 White cannot play 5 Kf6 without
losing the rook, while 5 Rf4 Ne8 6 Rf3
merely lengthens the solution. The
continuation after 4 Rf3 Ng7 5 Kf6
Nh5+ 6 Kf5 Ng7+ 7 Ke5 Kh7 we
already know from the *Oesterreichische
Schachrundschau* study.

Thus far, everything also works on the
ordinary 8 x 8 board. However, after
4...Nc7 White has no win on the normal
board. On a board from which the a-file
has been removed, we have **5 Kf6 Kf8
6 Rf2/Rf1 Ke8 7 Rd2 Nb5 8 Ke6 Nc7+
9 Kd6 Kd8 10 Rd3** and White wins, for
example **10...Ne8+ 11 Ke6+ Kc8
12 Ke7 Nc7 13 Kd6** and we have the
same winning position on the queen's
side as we had after 4 Rf3 Ng7 5 Kf6 on
the king's.

[Readers who are going through this
book with the aid of a computer will find
it very instructive to play through these
moves on the standard 8 x 8 board and
see just where Black needs access to the
a-file in order to draw. The computer
adds one further line, which echoes the
line 2...Nc4 3 Rd4 Nb6 4 Rd6 Nc4 5 Re6
and which Mandler may have thought
too obvious to mention: 2...Ne4 3 Rd5
Nc3 4 Re5, and the non-existent 4...Na4
is needed in order to draw.]

The Amelung position on rank
and file

2.8 (S343)
Wiener Schachzeitung 1925
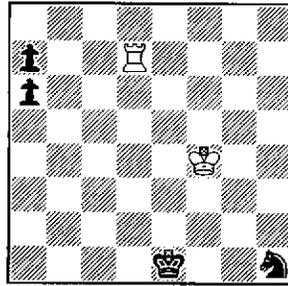
White to move and win

This study does not strictly belong to the
present chapter in terms of material, but
thematically it is very much in place.

1 Re7+ Kf2 (1...Kf1 2 Ke3 leads to a
shorter solution) **2 Rh7 Kg2 3 Rg7+**
(White dare not lose a tempo in case the
Black pawn finally arrives too soon,
hence the need for so many apparently
superfluous moves by the rook) **Kf2
4 Rg5** (getting into place for the
Amelung position) **a5 5 Rh5 Kg2 6 Ke3**.
Now we see why the rook had to come to
the fifth rank. Were it elsewhere, Black
could now save himself by 6...Ng3; but as
it is, 7 Rg5 would give the Amelung
position (see **2.2a**) and White would win.
6...a4 7 Rg5+ (not 7 Rd5 at once,
because of 7...a3 and White would arrive
too late) **Kf1 8 Rd5** (preparing the
Amelung position for the second time,
this time with the knight pinned on the
rank, and now Black has no time for ...a3
because White threatens 9 Kf3) **Kg2**
(8...a3 9 Kf3 Ke1 10 Re5+ Kd1/Kd2 11
Rc5 etc, 8...Ke1 9 Rc5 Kd1 10 Kf3)
9 Rd2+ Kg3 (9...Kh3 10 Kf4) **10 Rd1
Kg2 11 Kf4 Nf2 12 Rd2** and wins.

Knowing the preceding studies simplifies the solution of this twin

2.9 (S345)
Československý šach 1933
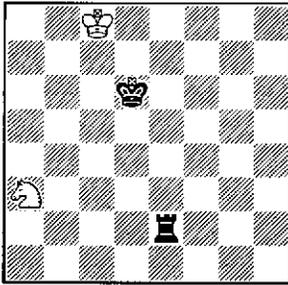
White to move and draw
(a) as set, (b) bRe2 to e1

(a) With the rook on e2, the first move is **1 Nb5+**. In order to win, Black would have to play 1...Kc6, and this is not possible on account of 2 Nd4+. 1...Kc5 is not good enough: 2 Nc7 Kb6 3 Nd5+ Kc6 4 Nb4+ Kc5 5 Nd3+. However, 1 Nc4+? would fail on account of 1...Kc5 2 Na5 Rc2 3 Nb7+ Kc6, giving a position already known from the *Oesterreichische Schachrundschau* study.

(b) Now **1 Nc4+** is the move that draws, for example 1...Kc5 2 Na5 Kb6 3 Nb7 (3 Nc4+? Kb5 4 Nd6+ Kc6 and Black wins) Rc1+ 4 Kb8 Kc6 5 Na5+ and Black cannot play 5...Kc5 on account of 6 Nb3+. Again, this position is already known to us, and without this knowledge the solution of the present study would be difficult. 1 Nb5+? fails against 1...Kc6 2 Nd4+ Kc5 3 Nb3+ Kb6 4 Nd4 Rc1+ 5 Kb8 Rd1.

Another twin in which the rook is shifted one square

2.10 (S346)
Československý šach 1933
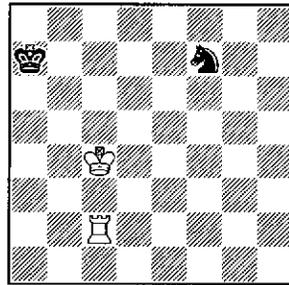
White to move and win
(a) as set, (b) wRe2 to c3

(a) Here there are a host of tries. 1 Kd5? Kb7! (1...Kb6/Nd8? 2 Rc8!) 2 Ke6 Ng5+ 3 Kf6 Nf3 and Black draws, or 2 Rf2 Nd8 3 Kd6 Kb6! 4 Rb2+ Ka6 5 Kc7 Ne6+ and White is getting nowhere. This is a typical line of play by Black. Black also meets 1 Rd2 by 1...Kb6, 1 Re2 by 1...Nd8 with either 2 Re8 (2 Kc5 Kb7 3 Kd6 Kb6) Nb7/Nc6 or 2 Rd2 Nc6, and 1 Rf2 by 1...Nd8 2 Rf8 Ne6 etc. Correct is **1 Rg2 Kb6 2 Rg6+ Ka5 3 Kd5** and wins.

(b) 1 Rg3? Nd6+ 2 Kd5 (the position of the White rook on g3 prevents 2 Kc5) Nb5 3 Kc5 Ka6 and White cannot check on the a-file. The move of the rook to the third rank has proved doubly unfortunate. The way to the win is now **1 Kd5 Kb7 2 Ke6 Ng5+ 3 Kd7! Kb6 4 Re3** etc.

In these two studies, it has been the rook whose displacement by one square has created the twin settings. That a similar displacement of the knight should lead to a complete change in the solution wuold hardly be remarkable.

[Mandler actually seeks to refute 1 Rg3 in (b) by playing 1...Nd6+ 2 Kd5 Nc8 3 Kc6 Ne7+ 4 Kd7 Nd5 'etc', but

the computer continues 5 Rb3 and captures the knight on move 22 at the latest. But 2...Nb5 does hold the draw, so the twin can stand with slightly different analysis. Would Mandler have accepted it in this modified form? We cannot be certain, but I think he probably would.]

Sometimes the knight finds itself far from its king, yet the stronger side cannot win

2.11 (S347)
Československý šach 1934
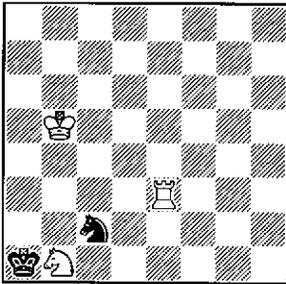
White to move and win

Where should the threatened rook go? It must stay on the e-file, lest the knight escape. For example, if White were to play 1 Rh3? there would follow 1...Kxb1 2 Kc4 Kb2 and White would be unable to win, because the rook would be unable to reach the Amelung position by playing to e2. 1 Re2 and 1 Rb3 allow 1...Nd4+. A superficial consideration might lead the solver to consider 1 Re4 as the answer. This does indeed work after 1...Kxb1 2 Kc4 Kb2 3 Re2, but Black has a better defence in 2...Kc1. Now 3 Kb3 does not help, because the Black king escapes to the d-file and White has no rook check at his disposal.

Correct is **1 Re5 Kxb1 2 Kc4 Kc1 3 Kb3 Nd4+ 4 Kc3** with an easy win. The object of 1 Re5 is to prevent 4...Nb5+.

But is this really the only way to win?

Would not 1 Re7 or Re8 be good enough? No, not 1 Re8, because the rook is lost after 4...Nb5+ 5 Kc4, and neither 5 Kb4 (5...Nd4!) nor 5 Kd3 (5...Na3) is good.

Not so transparent is the refutation of 1 Re7. It sometimes happens that the stronger side cannot win even though the knight has been driven far from its king. This is a case in point: 1 Re7? Kxb1 2 Kc4 Kc1 3 Kb3 Nd4+ 4 Kc3 Nb5+ 5 Kc4 Nd6+ 6 Kb3 (6 Kd3 Nb5) Nf5 7 Rc5 Nh4! and in spite of the apparently unfavourable position of the knight Black can hold the draw.

[The computer gives 7...Ng3 as an alternative draw in the last line, but it hardly detracts from the point Mandler is making.]

The merry capering of the hobby-horse

2.12 (S348)
by Richard Réti
Tidskrift för Schack 1929
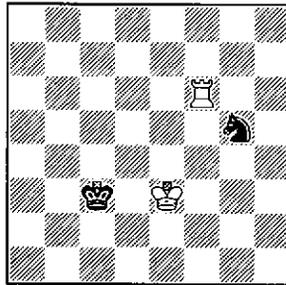
White to move and win

What is a Réti study doing in a collection of my compositions? The ending R v N has been largely ignored in the textbooks, and we cannot solve the studies in this chapter without a knowledge of certain fundamental positions. We have already seen several of them, and this beautiful Réti study is another. It prepares the ground for the following studies, within which it is wholly or partly contained.

Studies and endgame theory are not antitheses, studies are the building material of theory.

The Black king occupies a favourable square. If it were on b3, White would have an easy win: 1 Kf4 Nh3+ 2 Kg4 Ng1 3 Rf2 and 4 Rg2. The square c2 would also be bad, allowing White two possible ways of winning: 1 Rg6 Nf7 2 Kd4 Nd8 3 Kd5 Nb7 4 Ra6 or 1 Rf5 Ne6 2 Re5. White must proceed more carefully if the king is on c4. Now the way to win is 1 Kf4 Nh3+ 2 Ke4 (if the Black king were on c3, he would have a draw here by 2...Ke2) Ng5+ (2...Kc5 3 Ke3 and 4 Rg6) 3 Kc5 Nh3 (3...Kd3/Kc5 4 Rf5) 4 Rf3 Ng5 (4...Ng1 5 Re3) 5 Rf4+ and 6 Rf5.

So if we can expel the king from c3, we shall have our win. We will proceed thus: **1 Kf4 Nh3+ 2 Kf3** (threat 3 Rg6) **Ng5+ 3 Ke3!** This has brought us back to the starting position with Black to move. The knight can move neither to h3 (4 Rg6) nor to h7 (4 Rf5), so it is the king which must give way: **3...Kc4 4 Kf4 Nh3+ 5 Ke4 Ng5+ 6 Ke5 Nh3 7 Rf3 Ng5 8 Rf4+** followed by **9 Rf5** and White wins.

The hobby-horse capers merrily between g5 and h3, and White must proceed carefully and with forethought in order to catch him. Twice he plays so as to transfer the burden of moving to Black.

[In his text to this study, Mandler uses the affectionate diminutive *koníček* ("little horse") for the knight, instead of the normal word *jezdec* that he employs elsewhere. My rendering "hobby-horse" may be a translator's artefact, because I am not sufficiently familiar with early 20th-century Central European folk dance traditions to know whether he genuinely had something of this sort in mind, but the vision of an English Morris dance, with the hobby capering merrily on the outskirts, was irresistible.

An analytic note from the computer:

right at the end, 8 Rf5 is playable and indeed slightly simpler (8...Nh3 9 Ke4, with Ke3 and Rh5 to follow). The rest is impeccable. The same note applies to the next study.]

An even longer caper by the knight

2.13 (S349)
Československý šach 1933
(after R. Réti)
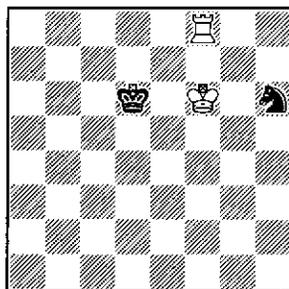
White to move and win

Here we lengthen the knight's pendulum. This is no more than an extension by five moves of the preceding study, since after Black's fifth move we have the position of the Réti study reflected through 180 degrees.

1 Rd8+ Kc5 2 Rd3 Ng4+ 3 Kf5 Nh6+ 4 Ke6 Ng4 5 Rf3 Kc6 6 Kf5 Nh6+ 7 Kf6 Ng4+ 8 Ke6 Kc5 9 Kf5 Nh6+ 10 Ke5 Ng4+ 11 Ke4 Nh6 12 Rf6 Ng4 13 Rf5+ Kc4 14 Rf4 Nh6 15 Ke5+ and White wins.

An unexpected encounter with the Réti study

2.14 (S350)
Československý šach 1933
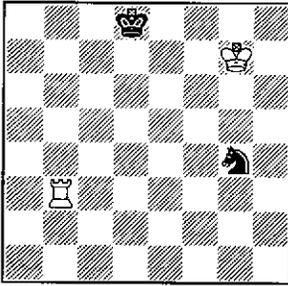
White to move and win

Our experience in the examples up to now has been that the king should attack the opposing knight diagonally in order to limit its powers of movement, even though this allows the knight to give check. The present study forms an exception. Here the king attacks the importunate knight laterally: **1 Kf7 Ne5+ 2 Ke6 Ng6 3 Kf6 Nf4 4 Kf5 Nh5 5 Rb7** etc.

But in this study we also have another unexpected encounter with the position of the Réti study: (**1 Kf7**) **Kd7 2 Rd3+ Kc6 3 Ke6 Kc5 4 Rf3 Kc6**. We already know the rest.

Two more occurrences of familiar manoeuvres

2.15 (S351)
Československý šach 1934
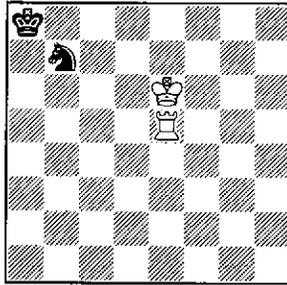
White to move and win
(a) as set, (b) wK to f7, wR to f6

(a) There is a dual at the end of the *Oest. Schachrundschau* study **2.3**. Instead of 9 Rg5, White can proceed otherwise, though the play is very complicated. Here this dual is removed, though of course at the cost of also removing the main line of the study. The solution is quite short, and goes **1 Kd7 Ka7 2 Kc8! Nd6+ 3 Kc7.** 1 Kd5? Ka7!

(b) **1 Ke6** (1 Ke7? Ka7) **Kb8 2 Kd7 Ka7 3 Kc6 Nd8+ 4 Kd6 Nb7+ 5 Kd5 Na5 6 Kc5 Nb7+ 7 Kb5** etc; 1...Na5 2 Rf5 (2 Kd5? Kb7!) Nc4 3 Rf4 Nb6 4 Kd6/Rb4 etc; 1...Nc5+ 2 Kd5 Nd7 3 Rf7 etc; 1...Ka7 2 Kd5. Here we have a manoeuvre from study **2.3** combined with the Amelung position.

A position which prompted a systematic investigation

2.16 (S352)
Československý šach 1946
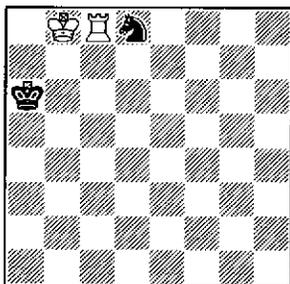
Shift the position bodily so that
Black to move can draw

How can we place this configuration on the board so that Black to move can draw? This can be answered only by a laborious consideration of all 30 possible settings. I will therefore reveal at once the answer which came out of my researches. The unique way to set this configuration on the board so that Black to move can draw is **White Kd7, Re7; Black Kc5, Nf7**. Black can now hold out by **1...Nh6 2 Rh7 Ng4 3 Rh5+ Kb6 4 Kd6 Ne3**.

The solver does not have to take my word for it. Why should precisely this position be drawn, and not a position one or more squares away from it? How should Black continue, if White plays **5 Rc5**?

This question gave rise to further research. The position that arises is so important for the ending R v N that we must get to grips with it. Without it, nothing is simple.

[The computer indicates an alternative draw by 4...Nf2 (5 Kd5 Ng4 6 Kd4 Kc7 and Black will eventually regroup), but 4...Ne3 is the simpler and more natural move and I don't think the existence of this alternative takes away from what follows.]

Three instructive diagrams

2.17 (S353)
Československý šach 1946-47
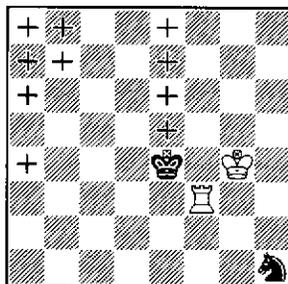
A - White to move wins
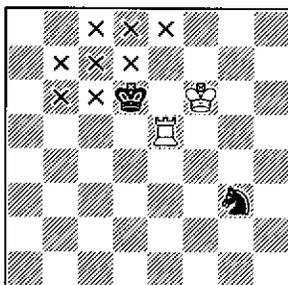
B - Black to move draws
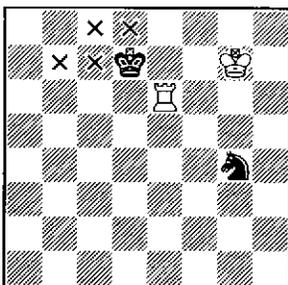
C - White to move, Black draws

The three diagrams **A-C** show positions which can be reached from the preceding diagram, and which arise very frequently in the analysis of endings with R v N.

White to move wins in **A**, or if the

position is shifted so that the Black king is on any square marked +.

B shows the same configuration with Black to move. Black draws in the position shown, or if it is shifted so that the Black king is on any square marked ×.

In **C**, the White king is one square further away from the Black. If White is to move, Black draws in the position shown, or if it is shifted so that the Black king is on any square marked ×.

I would have to present extensive analysis to prove the correctness of these diagrams, and it would demand far too much space.

We can now complete the analysis of diagram **2.16**. We stopped with White Kd6, Rc5, Black Kb6, Ne3, Black to play move 5. We see from diagram **2.17B** that White cannot win. The reason is that after **5...Nd1 6 Rc1 Ne3 7 Rc3 Nf5+ 8 Ke6 Nd4+ 9 Kd5 Nb5** we have a position similar to the Amelung position but one rank lower, and this enables Black to hold the draw.

[The definitive computer results now available differ from Mandler's only in showing a very difficult win in diagram **A** with the Black king on a6: with best play, White can capture the knight on move 22. I have adjusted Mandler's diagram accordingly. The alteration appears not to affect the exposition of subsequent studies.]

Four more related diagrams...

2.18 (S354)
Československý šach 1933
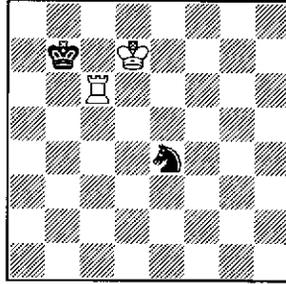
White to move throughout
(a) as set, White wins
(b) wK to e7, Black draws
(c) as (a) down a rank, Black draws
(d) as (b) down a rank, White wins

This is a collection of chess compositions; it is concerned with theoretical analyses only in so far as these are useful for understanding solutions. We shall therefore engage in only a few positions deserving of special attention - and these four positions are cases in point. Most of the remaining positions in diagrams **2.17A-C** are more easily mastered.

Usually, twin compositions feature different solutions leading to the same result. These four diagrams show something quite different. If we shift position (a) down a rank, we obtain position (b), but White wins in one case and Black holds the draw in the other. The reverse happens if we shift position (b) down a rank. And positions (a) and (b) also form a pair of twins, as do (c) and (d), and once more everything is reversed; White wins in one part, Black draws in the other.

In this first diagram, **1 Rc4 Nf6+ 2 Ke7 Nd5+ 3 Kd6 Nb6 4 Rb4** brings us to the now familiar Amelung position. If instead **1...Ng5 2 Rf4 Kb6**, the move **3 Ke7?** would be a decisive mistake.

Correct is **3 Kd6** with continuation **3...Nh3 4 Rf3 Ng5 5 Re3**.

Somewhat more complicated is **1...Ng3**. There are several ways to win, but also some tempting moves which analysis shows to be faulty. Thus for example after **2 Ke6 Kb6 3 Ke5 Kb5** White must not play 4 Kd5 on account of 4...Ne2, as we can see from **2.17A**. Correct is **4 Rc2 Kb4 5 Rg2 Nf1 6 Kd4** etc. On **1...Nf2** we play **2 Rd4 Kb6** and after **3 Ke6 Kc5 4 Ke5** we reach one of the winning positions shown in **2.17B**. However, if Black plays **3...Kb5**, we must not automatically play 4 Ke5, which would let Black draw (4...Kc5, see **2.17A**). Correct is **4 Kf5**. After **1...Nd2** the moves **2 Rb4+ Ka6 3 Kc6 Ka5 4 Kc5** lead to a win shown in **2.17B**, but wrong would be 2 Rd4 Nf3 3 Rd5 Kb6 4 Ke6 Kc6 as shown in **2.17A**. The repeated references to these auxiliary diagrams show their indispensable nature for resolving endings with R v N.
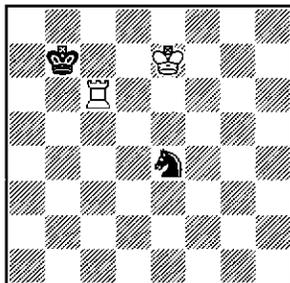
(b) White to move, Black draws

The same position will arise in the analysis of (b). Here, after **1 Rc4 Nd2** the move 2 Rb4+ forfeits any chance of winning, and the continuation **2 Rd4 Nf3 3 Rd5 Kc6 4 Ke6** leads only to **2.17B**; Black draws by **4...Ne1**.

In (c) and (d), we have the same positions down a rank. Now everything is changed. In (c), after **1 Rc3**, Black can draw by **1...Nf5+**. Conversely, Black was able to draw in (b) only by playing ...Ne1

and in (d) the equivalent move is no longer available, so White wins by **1 Rc3 Nd1 2 Rd3 Nf2 3 Rd4**.
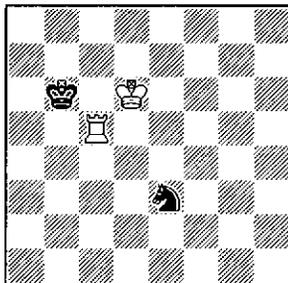
(c) White to move, Black draws
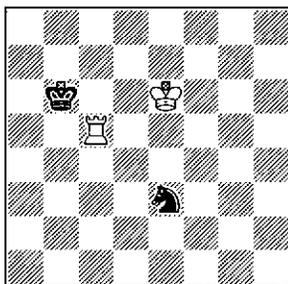
(d) White to move and win

[The computer has only one comment: in (a), in the line 1...Nd2 2 Rd4 Nf3, White has a difficult win by 3 Rd1, and it is 3 Rd5 rather than 2 Rd4 which is the decisive mistake.]

...and a twin study originating from them

*2.19 (S355)
Československý šach 1933
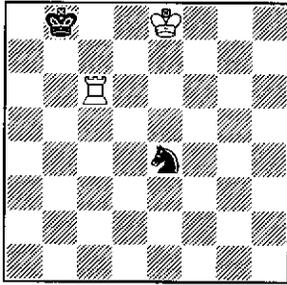
White to move and win
(a) as set, (b) everything one rank down

The solution flows automatically from the preceding four diagrams. In (a), **1 Kd7 Kb7** gives the winning position shown in part (a) of **2.18**, whereas 1 Ke7 Kb7 brings us to the drawn position of **2.18** (b).

In (b), everything is the other way round, and now White must play **1 Ke6**.

An unusually complicated affair

2.20 (S356)
Československý šach 1946
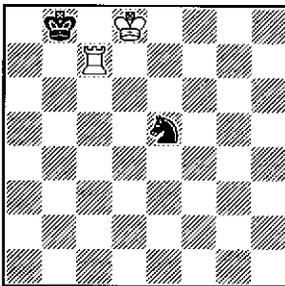
White to move and win

Let us imagine that we reach this position in a game. I might almost say that it cannot be completely analysed without the aid of our three auxiliary diagrams **2.17A-C**. We keep coming back to a shifted version of the original position. This dependence is even closer than might appear from playing through the solution, for in analysing individual positions of this kind, the solver who conscientiously examines every possibility will continuously find new positions of the same kind, even though not as many as arise from **2.20**.

First, some tries. 1 Rc5! (this is correct) Nf3 2 Rb5+? Ka7 3 Kc7 Ka6 4 Rd5 Ne1 and draws; 2 Rd5? Kb7 3 Kd7 Kb6 4 Kd6 Ne1; 2 Rc3! Nd4 3 Rc4! Nf3 4 Rb4+? Ka7 5 Kc7 Ka6 6 Kd6 Ka5 7 Kc5 Nd2 and we have **2.17A**.

Now the solution. **1 Rc5 Nf3** (1...Nd3 see line 8 below) **2 Rc3 Nd4** (2...Ne5 see line 6, 2...Nd2 line 7) **3 Rc4 Nf3** (3...Ne2 see line 4, 3...Nf5+ line 5) **4 Ke7 Kb7** (4...Ne5 see line 1) **5 Kd6 Kb6 6 Kd5 Kb5 7 Rc2 Ne1 8 Rc3 Kb4 9 Kd4** and wins (**2.17B**).

1) 4...Ne5 5 Rc5 Nf3 (5...Nd3 see line 2) 6 Rd5 Kc7 7 Kf6 Kc6 and White wins (**2.17C**), but not 7 Ke6? Kc6 and Black draws (**2.17A**); if 7...Kb6 then 8 Kf5 and 8 Rd3 both win.

2) 5...Nd3 6 Rd5 Nf4 (6...Nf2 see line 3) 7 Rd4 (7 Rd6? Kc7 and draws, **2.17A**) Ne2 8 Rc4 Kb7 9 Ke6 Kb6 10 Ke5 (10 Kd5? Kb5 draws, **2.17A**) Kb5 and White wins (**2.17C**).

3) 6...Nf2 7 Rd4 Kc7 8 Ke6 Kb6 9 Kf5 (9 Ke5? Kc5 draws, **2.17A**) Kc5 and White wins (**2.17C**).

4) 3...Ne2 4 Kd7 Kb7 5 Kd6 Kb6 6 Ke5 (6 Kd5? Kb5 draw, **2.17A**) Kb5 and White wins (**2.17C**).

5) 3...Nf5 4 Kd7 Ne3 5 Rc5 Kb7 6 Ke6 (of course not 6 Kd6) Kb6 and White wins (**2.17C**).

6) 2...Ne5 3 Ke7 Kb7 4 Kd6 Nf7+ 5 Ke6 Ng5+ 6 Kd7 Nc4 7 Rc4 and wins (**2.17A**, after playing Rc4).

7) 2...Nd2 [see note at end].

8) 1...Nd3 2 Rb5+ Ka7 3 Kc7 Ka6 4 Kc6 and wins **(2.17B)**.

[The computer sometimes shortens the solution by deviating from Mandler's systematic treatment, but this is hardly important. There is however an error in Mandler's variation 7. He gives 3 Kd7 Kb7 4 Ke6 Ne4 5 Rc4 Ng5+ 6 Kd7 intending 6...Nf3 7 Kd6 Nd2 8 Rb4+ Ka6 9 Kd5 Ka5 with a win by diagram **2.17C**, but the computer prefers 6...Kb6 and in fact 3 Kd7 forfeits the win; White must play Rc7 first, and Kd7 the move after. The win after 3 Rc7 takes only 13 more moves and I am sure Mandler was aware of it (the few wins he missed were much deeper), but the play is complicated and it is not clear how he would have chosen to present it had he realised that it was needed.]

A conventional twin

2.21 (S357)
Československý šach 1946
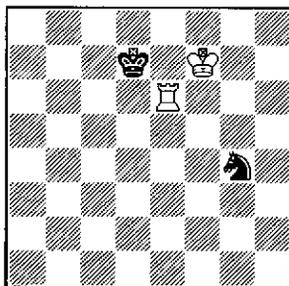
Black to move and draw
(a) as set, (b) everything one rank higher

These positions have the same stipulation, Black to move and draw, but the routes to the draw differ.

(a) **1...Nf2 2 Re2 Nh3** and either **3 Kf6 Nf4 4 Rd2+ Kc6 5 Ke5 Ng6+ 6 Kf5 Nf8** and draws or **3 Rd2+ Kc6 4 Kf6 Nf4 5 Ke5 Ng6+**; 1...Nh2? 2 Kf6!; 1...Kc7 2 Re4 Nf2 3 Rd4 Kc6 4 Ke6 Kc5

5 Ke5 **(2.17B)**.

(b) **1...Kc8 2 Re5 Nf3 3 Rd5 Kc7 4 Ke7 Kc6 5 Ke6 (2.17B) Ne1** and draws; 1...Nf3? 2 Re3 Nh4 (2...Ng5 3 Kg7 Kd7 4 Kf6 Nh7+ 5 Kg6 Nf8+ 6 Kf7 etc) 3 Kf7 Nf5 4 Rd3+ Kc7 5 Ke6 Ng7+ 6 Kf6 Nh5+ 7 Ke7 Nf4 8 Rc3+ Kb6 9 Kd6 and either 9...Kb5 10 Re3 or 9...Nc2 10 Rc4 Kb5 11 Kd5.

[Valid alternatives: in (a), 2...Ng4, 6...Ne7+; in (b), 3...Kb7/Ne1/Ng1, 4...Kb6/Ne1/Ng1.]

A study in systematic movement

2.22 (S358)
Československý šach 1946
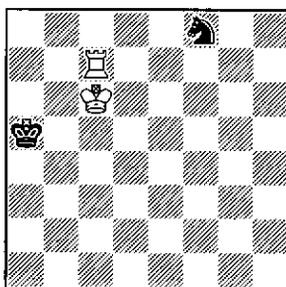
Black to move, White wins

Here again, we shall largely be dependent on our auxiliary diagrams. **1...Ne6 2 Rd7 Kb4 3 Rd6 Ng7** (3...Ng5 see line 3) **4 Kd5 Nf5 5 Re6 Kc3 6 Re5 Nh6** (6...Nh4 see line 1). So far, everything has proceeded in a pleasantly systematic manner, but now 7 Ke4 would be a mistake (7...Nf7 8 Rd5 Kc4 and draws). Correct is **7 Rg5**.

1) 6...Nh4 7 Rh5 Ng2 (7...Ng6 see line 2) 8 Rh3+ Kb4 9 Kd4 Ne1 10 Rc3.

2) 7...Ng6 8 Ke4 Ne7 9 Rh7, or 8...Nf8 9 Rh6 and either 9...Kc4 10 Rd6 Kc5 11 Ke5 or 9...Nd7 10 Kd5 Kb4 11 Rd6.

3) 3...Ng5 4 Rg6 Nf3 (4...Nf7 5 Kd5) 5 Rg4+ Ka5 6 Kc5 Nd2 (6...Ne1 7 Rg3) 7 Rb4.