Chapter 8

Three coin puzzles

The first two of these, though amusing, are mathematically trivial, and would have been discovered by anybody who cared to look for them. Very probably somebody did, though nobody claimed to have done so when they appeared in *The Games and Puzzles Journal* a few years ago. The third is of somewhat heavier metal.

8.1 (once, twice, four times ...)

To get four sums of money from coins of value 1, 2, 5, 10, 20, 50, and 100, each consisting of two coins and each being twice as large as its predecessor, set

15 = 5 + 10
30 = 10 + 20
60 = 10 + 50
120 = 20 + 100

8.2 (one potato, two potatoes ...)

To obtain a sum of money which can be realised by one, two, three, four, or five of the above coins, consider

20
10 + 10
10 + 5 + 5
5 + 5 + 5 + 5
10 + 5 + 2 + 2 + 1

This was the only solution with the English coinage as it was some years ago, but now the existence of a £2 coin provides an alternative.

8.3 (11 coins, ≤ 2 duds, 5 weighings)

Given eleven coins of which at most two are duds, and given that (a) a dud is either heavy or light, (b) two heavy duds or two light duds balance each other, and (c) a heavy dud and a light dud balance two true coins, the duds if any can be identified in five weighings by balancing

1, 3, 4 against 2, 5, 6
1, 4, 2 against 3, 7, 8
1, 2, 3 against 4, 9, 10
5, 7, 9 against 6, 8, 10

and setting up a fifth weighing in the light of the results. It may be verified that each of the 81 possible sets of results from these four weighings can arise in precisely three ways.

This was a fortunate arithmetical accident. If we have n coins, there are $2n^2 + 1$ possibilities, and w weighings give 3^w different sets of results. $2.11^2 + 1 = 3^5$, and a weighing of three coins out of eleven against three gives $81 = 3^4$ possibilities whether the result is left down, right down, or balance.